KINEMATICS ANALYSIS OF A PARALLEL ROBOT WITH A PASSIVE SEGMENT

ANÁLISIS DE LA CINEMÁTICA DE UN ROBOT PARALELO CON UN SEGMENTO PASIVO

Abdelhakim Cherfia¹ Abdelouahab Zaatri¹ Max Giordano²

Recibido 12 de diciembre de 2005, aceptado 13 de abril de 2007 Received: December 12, 2005 Accepted: April 13, 2007

RESUMEN

Este trabajo presenta el modelo geométrico de un robot paralelo con tres grados de libertad (d.o.f) agregados a un segmento central pasivo del PPP. Esta estructura proporciona un movimiento de traslación pura. También determinaremos las relaciones entre las velocidades generalizadas y articulares usando la matriz Jacobiana inversa. Además, determinamos las relaciones recíprocas entre las velocidades cartesianas y angulares del end-effector vía velocidades articulares por la derivación simple de las expresiones del modelo geométrico directo. Una determinación del espacio de trabajo basado en el análisis del modelo geométrico es derivado seguido por un cálculo numérico de todos los puntos que deben alcanzarse permitiendo una visualización gráfica de tal espacio de trabajo. Por otra parte, el análisis de los coeficientes de la matriz Jacobiana permite asegurar que no haya singularidades del tipo 1 y 2 en tal estructura. Se ha realizado un prototipo de robot paralelo en nuestro laboratorio para validar los modelos propuestos.

Palabras clave: Modelo geométrico, manipulador paralelo, segmento pasivo, matriz Jacobiana, espacio de trabajo, singularidad.

ABSTRACT

This paper presents a geometrical model of a constrained robot of three degrees of freedom (d.o.f) added to a PPP passive central segment. This structure provides a pure translation motion. We will also determine the relations between generalized and articular velocities by using the inverse Jacobian matrix. Further, we determine the reciprocal relations between cartesian and angular velocities of the end-effector via articular velocities by simple derivation of the direct geometrical model expressions. A determination of the workspace based on the geometrical model analysis is derived followed by a numerical calculation of all the atteignables points enabling a graphical visualisation of such a workspace. Moreover, the analysis of the Jacobian matrix has permitted to ensure that there are no singularities of type 1 and 2 in such a structure. A prototype of a parallel robot has been built up in our laboratory in order to validate the proposed models.

Keywords: Geometrical model, parallel manipulator, passive segment, Jacobian matrix, workspace, singularity.

INTRODUCTION

These last years, the manipulators with a parallel structure have constituted a pole of interest for many researchers and industrialists because of their interesting performances: high load capacity, movements at high rate, high mechanical rigidity, low mobile mass, simplicity of mechanical engineering and possibility of locating the actuators and the sensors on the base, or close to the base.

Historically, these mechanical systems were initially developed for applications other than the pure robotics field. The first mechanical system of this type was the machine built by Gough [1] in 1949 to test the tires of

¹ Mechanical Engineering Department. Mentouri University. Road Ain El Bey 25000. Constantine, Algeria. E-mail: cherfia_abdelhakim@yahoo.fr

² LMECA, ESIA - Savoie University BP 806, 74016. Annecy Cedex France. E-mail: max.giordano@esia.univ-savoie.fr

the planes. Thereafter, Stewart [2] used this structure as a flight simulator. However, the idea to use parallel structures in robotics is in fact more recent. Indeed, Hunt [3] has identified certain parallel architectures that offer more significant potential in robotics. As a consequence, several researchers [4-11] have deeply analysed these architectures.

In this context, a special class of parallel robot architecture, with which we are concerned in this paper, has more recently emerged. This class is characterized by the addition of some constraints and is actually gaining a great attention [12-14]. These constraints are due to the addition of some passive segments into the classical parallel robot architecture. This class of parallel robots reduces the number of solutions which may be numerous in non constrained parallel robots, leading to the difficulty of determining the solution of the geometrical model. The addition of constraints also enables to determine the orientation and the position parameters or variables of the end-effector. One remarkable study concerning the determination of the geometrical analysis of a six dof constrained parallel robot has been carried out by Joshi and Tsai [13].

Beside the geometrical analysis of parallel robot, workspace and manipulability constitute important indexes for evaluating the performance of robot manipulators. How to determine the workspace and singularities are among the most relevant issues in the design of robot manipulators. This fact is particularly sensitive for parallel manipulators which are known to have relatively small useful workspaces.

Despite the complexity of parallel structures, many authors have used geometric analysis in order to determine the workspace for parallel manipulators [15,16]. A classical approach consists of simplifying the problem by fixing a certain number of degrees of freedom until only three degrees of freedom are left free [17-20]. This is usually done by the fixing of the platform orientation. Translational parallel manipulators have also benefited from such studies by many researchers [21-23]. Along this, a lot of studies have been devoted for the determination of parallel robot workspaces, their optimisation and the analysis of singularities. Geometrical workspace determination has been performed [24]. Numerical determination of the all atteignables points contained in the workspace has been carried out in [25-28].

In this paper, we are concerned with the study of a particular constrained parallel robot architecture. This structure is constituted of a parallel structure of three d.o.f and four segments with one passive central segment of type PPP to constrain the structure to move in a pure translation.

After the description of our mechanical system, we present the establishment of the equations corresponding to the inverse and direct geometrical models. We also present a visualisation of the workspace by numerical computation of all attainable points. Then, we present the kinematics modelling of our structure. Next, we present the experimental prototype we have designed and build up in order to validate the theoretical analysis we have developed.

DESCRIPTION OF THE MECHANICAL SYSTEM

The manipulator under consideration is of three d.o.f. It consists of a mobile platform connected to a fixed platform via three active segments and one passive central segment. One calls segment each open chain that connects the base to the mobile platform (figure 1).



Figure 1. Parallel robot with four segments.

The three active bars are connected to the fixed base by universal joints and to the mobile base by kneecap joints connections with three dof. The passive central bar with three dof can have several mechanical structures but we choose a structure of PPP type to give the end-effector manipulator pure translation movements.

The analysis of the system structure implies that for the system to be in balance and can be controlled, it requires that the mobility of the mechanism becomes null when the actuators are blocked in a given configuration. The mobility "m" of an isostatic space mechanism is given by Grûbler formula [4]

m = 6(L-n-1) +
$$\sum_{i=1}^{n} d_i$$
 (1)

With:

- L : the total number of solids including the base.
- $N \;$: the total number of articulations.
- d_i: the number of degrees of freedom of articulation i.

In the case of a parallel structure, if we note by p: the number of segments and by Ck: the number of dof associated with each segment, and then we have:

$$\sum_{i=1}^{p} Ck = m + 6(p-1)$$
(2)

By developing the relation (2) we obtain the following expression: C1 + C2 + C3 + C4 = 21.

We note by C1 the number of dof associated to the passive segment and by C2, C3, and C4 those associated to the active segments. For our particular structure, we choose: C1=3 and C2=C3=C4=2+1+3.

C1=3 means that we impose 3 dof for the passive segment which are the three translations. On the other hand, for the active segments (C2, C3, C4), we impose a universal joint, a slide and a kneecap joint. The slide is an active axis while the others are passive.

GEOMETRICAL MODEL

Let us consider a reference frame X, Y, Z of center O related to the fixed platform and a reference frame U, V, W of center P related to the mobile platform (figure 1).

The active segments are related to the fixed platform at points A_1 , A_2 , A_3 which are located at a distance ra with respect to the center O and connected to the mobile platform at the points B_1 , B_2 and B_3 located at a distance rb with respect to the center P of the mobile platform.

The angle β_i is measured between axis X and line OA_i and also between the axis U and the line PB_i.

We introduce the matrix ${}^{A}T_{B}$ that defines the orientation and the position of the mobile platform with respect to the basic reference frame. We have:

We choose the angle $\beta_1 = 0$.

The co-ordinates of the vectors OA_i and PB_i can be written in the following form:

$$\overline{{}^{A}OA_{i}} = {}^{A}a_{i} = \begin{bmatrix} a_{ix}, a_{iy}, & 0 \end{bmatrix}^{T}$$
(3)

$$\overline{{}^{B}PB}_{i} = {}^{B}b_{i} = \begin{bmatrix} b_{iu}, b_{iv}, 0 \end{bmatrix}^{T}$$
(4)

where:

$${}^{A}a_{i} = \left[raC\beta_{i}, raS\beta_{i}, 0\right]^{T}, i = 1, 2, 3$$

$$(5)$$

$${}^{B}b_{i} = \left[rbC\beta_{i}, rbS\beta_{i}, 0 \right]^{T}, i = 1, 2, 3$$
(6)

where: $C\beta_i$ and $S\beta_i$ are respectively the cosine and sine of angles β_i .

Let us consider the point of connection G of the passive segment to the fixed platform and the point of connection H of the same segment to the mobile platform.

The co-ordinates of these points of connection are given by:

$$\overline{{}^{A}OG} = {}^{A}g = \left[g_{x}, g_{y}, 0\right]^{T}$$
(7)

$$\overline{{}^{B}PH} = {}^{B}h = \begin{bmatrix} h_{u}, h_{v}, 0 \end{bmatrix}^{T}$$
(8)

Let's note by Px, P_{y} , Pz the co-ordinates of the point P which is the center of the (U,V,W) frame with respect to the basic reference frame.

If we consider the vector ^AP which is used to define the position of the mobile platform, then:

$${}^{4}P = \overline{OP} = \left[P_{x}, P_{y}, P\right]^{T}$$
(9)

the rotation matrix ${}^{A}R_{B}$ is used to define the orientations of the mobile platform with respect to the basic reference frame.

Let's note ψ , θ and ϕ the RTL angles defined by the succession of three rotations around three axes X, Y and Z to pass from the frame R_0 to R_1

The matrix ${}^{A}R_{B}$ can be thus written as follows:

$${}^{A}R_{B} = R_{z}(\phi) * R_{y}(\theta) * R_{x}(\psi)$$
(10)

143

$${}^{A}T_{B} = \begin{bmatrix} {}^{A}R_{B} & {}^{A}P^{A} \\ 1 & 0 \end{bmatrix}$$
(11)

The central segment can be analysed as an articulated mechanical system with an open chain. The equation of closure can be then written as follows:

$${}^{A}T_{0}{}^{0}T_{1}(\theta_{1}){}^{1}T_{2}(\theta_{2}){}^{2}T_{3}(\theta_{3}){}^{3}T_{B} = \begin{bmatrix} {}^{A}R_{B} & {}^{A}P \\ 1 & 0 \end{bmatrix}$$
(12)

By applying the equation of closure (12), we can have the expression that links the articular co-ordinates qi which represent the lengths of the active segments to the operational co-ordinates of the mobile platform center. This leads to the following equations:

$$\overline{Op} = \overline{OA_i} + \overline{A_i} \overline{B_i} + \overline{B_i} \overline{P}$$

$${}^{A}P - A_{ai} - q_i + \overline{PB_i} = 0$$

$$q_i = {}^{A}P + A_{bi} - A_{ai}$$

$$q_i = {}^{A}P + {}^{A}R_{B} {}^{B}b_i - A_{ai}$$

$$q_i = \left[{}^{A}P + {}^{A}R_{B} {}^{B}b_i - A_{ai} \right]^{T} * \left[{}^{A}P + {}^{A}R_{B} {}^{B}b_i - A_{ai} \right] (13)$$

The resolution of (12) by identification is obvious [13]:



With:

 L_i : the length of the passive segment for i = 1, 2, 3.

By developing the equation (13), we have the following relation:

$$q_{i}^{2} = P_{x}^{2} + P_{y}^{2} + P_{z}^{2} + r_{a}^{2} + r_{b}^{2} - 2r_{a}r_{b} - 2P_{x}r_{a}C\beta_{i}$$
$$+2P_{x}r_{b}C\beta_{i} - 2P_{y}r_{a}S\beta_{i} + 2P_{y}r_{b}S\beta_{i}$$
(14)

Where

$$q_i \pm \sqrt{\left[{}^{A}P + {}^{A}R_{B} {}^{B}b_i - A_{ai} \right]^T} \times \left[{}^{A}P + {}^{A}R_{B} {}^{B}b_i - A_{ai} \right]}$$
(15)

To simplify the form of theses expressions, we define:

$$e_{1} = q_{1}^{2} - q_{2}^{2}$$

$$e_{2} = q_{1}^{2} - q_{3}^{2}$$

$$e_{3} = q_{2}^{2} - q_{3}^{2}$$

$$e_{4} = 2r_{a} S\beta_{2} - 2r_{b} S\beta_{2}$$

$$e_{5} = 2r_{a} S\beta_{3} - 2r_{b} S\beta_{3}$$

$$e_{6} = -2r_{a} + 2r_{b}$$

$$e_{7} = 2r_{a} S\beta_{2} - 2r_{b} C\beta_{2}$$

$$e_{8} = 2r_{a} C\beta_{3} - 2r_{b} C\beta_{3}$$

$$e_{9} = r_{a}^{2} + r_{b}^{2} - 2r_{a} r_{b}$$

The lengths of the extensible segments can be expressed as follows:

$$q_{1}^{2} = P_{x}^{2} + P_{y}^{2} + P_{z}^{2} + e_{9} + P_{x} e_{6}$$

$$q_{2}^{2} = P_{x}^{2} + P_{y}^{2} + P_{z}^{2} + e_{9} - P_{x} e_{7} - P_{y} e_{6}$$

$$q_{3}^{2} = P_{x}^{2} + P_{y}^{2} + P_{z}^{2} + e_{9} - P_{x} e_{8} - P_{y} e_{5}$$
(16)

Equations (16) represent precisely the inverse geometrical model of our constrained robot. Therefore, given geometrical dimensions of the parallel robot, we can determine the lengths of the active segments which are necessary to move the mobile platform towards the point of the workspace located at co-ordinates (Px, Py, Pz)^T.

Expressions (16) have been implemented for controling the robot end-effector.

The direct geometrical model is used to pass from the articular co-ordinates to the operational co-ordinates expressed in the reference frame of the base.

By combining expressions (16), we can have:

$$q_1^2 - q_2^2 = P_x \left(e_6 + e_7 \right) + P_y e_4 \tag{17}$$

$$q_1^2 - q_3^2 = P_x \left(e_6 + e_8 \right) + P_y e_5 \tag{18}$$

The resolution of this system of equations enables to determine P_X , P_v , P_Z as follows:

$$P_{x} = \frac{e_{2} \cdot e_{4} \cdot e_{5} \cdot e_{1}}{e_{4}(e_{6} + e_{8}) - e_{5}(e_{6} + e_{7})}$$

$$P_{y} = \frac{e_{1} - P_{x}(e_{6} + e_{7})}{e_{4}}$$

$$P_{z} = \pm \sqrt{q_{1}^{2} - P_{x}^{2} - P_{y}^{2} - e_{9} - P_{x}e_{6}}$$
(19)

Ingeniare. Revista chilena de ingeniería, vol. 15 Nº 2, 2007

Equations (19) represent the direct geometrical model of our constrained parallel robot. They allow the passage from the articular co-ordinates to the configurations of the mobile platform.

Equations (19) have also been implemented for controling our robot.

WORKSPACE

The constrained parallel robot workspace under consideration can be determined numerically according to the direct geometrical model given by equations (19). By varying systematically the active segment lenghts between their minimal values q_{imin} and their maximum values q_{imax} , reachable positions of the mobile platform center (Px, Py, Pz) can be computed. The set of these positions provides the possibily to visualize the robot workspace. The following figure visualize the 3D robot workspace



Figure 2. Representation of the workspace in 3D space.

KINEMATICS MODEL

The inverse kinematics model establishes a relationship between the articular velocity and the operational velocity. The inverse Jacobian matrix establishes the relation between cartesian and angular velocity and articular velocity.

The inverse Jacobian matrix is given analytically by the relation:

$$\frac{dq}{dt} = J^{-1} \frac{dx}{dt}$$
(20)

By simple derivation of equations (16), we can deduce the inverse kinematic model and the inverse jacobian matrix which can be written in the following form:

$$J^{-1} = \begin{bmatrix} \frac{2P_x + e_6}{2q_1} & \frac{2P_y}{2q_1} & \frac{2P_z}{2q_1} \\ \frac{2P_x - e_7}{2q_2} & \frac{2P_y - e_4}{2q_2} & \frac{2P_z}{2q_2} \\ \frac{2P_x - e_8}{2q_3} & \frac{2P_y - e_5}{2q_3} & \frac{2P_z}{2q_3} \end{bmatrix}$$

The obtained inverse Jacobian matrix will be used for the study of the singular positions of the constrained parallel manipulator, for the evaluation of its maneuverability and also for the optimization of its architecture.

The direct kinematics model provides the relationships for determining cartesian and angular velocity of the end effector given the articular velocity. By simple derivation of expressions (19) we obtain the direct Jacobian matrix which is written:

$$J = \begin{bmatrix} \frac{q_1}{e_6 + e_7} & \frac{-q_2}{e_6 + e_7} & \frac{-q_3}{e_6 + e_7} \\ 0 & -\frac{q_2}{e_4} & \frac{q_3}{e_4} \\ \frac{q_1}{p_2} \begin{bmatrix} 1 - \frac{2p_x + e_6}{e_6 + e_7} \end{bmatrix} & \frac{q_2}{2p_2} \begin{bmatrix} (2p_x + e_6) + \frac{2p_y}{e_4} \end{bmatrix} & \frac{q_3}{2p_2} \begin{bmatrix} (2p_x + e_6) - \frac{2p_y}{e_4} \end{bmatrix} \end{bmatrix}$$

Singularities correspond to certain configurations of parallel manipulators which have to be avoided because they lead to an abrupt loss of manipulator rigidity. In the vicinity of these configurations, the manipulator can become uncontrolable and the articular forces can increase considerably with the risk to even damage the manipulator mechanisms.

Singularities of type 1 or parallel singularities correspond to the condition where det $(J^{-1}) = 0$. The kinematics analysis of our manipulator reveals that there are no singularities inside the workspace.

Singularities of type 2 or serial singularities correspond to the condition where det (J) = 0. If this situation arises, this means that the matrix J is degenerated and there is infinity of solutions for the inverse geometrical model in the vicinity of these points. For these singular positions, in the presence of the articular movements, the mobile platform will remain fixed and the manipulator will benefit of a great rigidity [21]. In our case, the analytical solution given by expressions (16) shows the unicity of the solution of the inverse geometrical model. This result confirms the non existence of the singularities of type 2.

EXPERIMENTAL PROTOTYPE AND NUMERICAL RESULTS

The experimental prototype of the parallel robot which has been built up in our laboratory is constituted of tack welded structure and D.C. motors as actuators (figure 3 and figure 4). A control based on the direct geometrical model as well as a control based on the inverse geometrical model have been implemented. We have also developed a graphical user interface which allows the user to interactively specify the articular or the Cartesian coordinates (figure 5 and figure 6).

We can conclude that the theoretical expressions concerning inverse and direct kinematics are well verified. In the practice, our results agree qualitatively with our theorical expressions.



Figure 3. Scheme of our parallel robot prototype.1. Fixed Base. 2. Mobile Platform. 3. Actuator.4. Passive segment. 5. Kneecap. 6. Ball and socket joint. 7. Place of the final body.

GEOMETRICAL CHARACTERISTICS

The prototype has the following geometrical data (see figure 4):

 $ra = 690 \text{ mm}, rb = 350 \text{ mm}, \beta_1 = 0^\circ, \beta_2 = 120^\circ \text{ et } \beta_3 = 240^\circ.$



Figure 4. Picture of our parallel robot.

| 🖾 COMMANDE DU ROBOT ROPAC | | | |
|----------------------------|-----------|---------|---------------------------|
| MODELE GEOMITRIQUE INVERSE | | | |
| | DONNEES : | | RESULTATS : |
| Px | 90 | q1 | 618.4254199173898 |
| Py | 45 | q2 | 650.2173402767120 |
| Pz | 605 | q3 | 674.9202915900514 |
| | | GEOMITE | EXECUTION BIQUE DIRECT |
| | DONNEES : | | RESULTATS : |
| q1 | | Pz | |
| q2 | | Py | |
| q3 | | Pz | |
| | CALCUL | | EXECUTION |

Figure 5. The control by the inverse geometrical model.



Figure 6. The control by direct geometrical model.

The points of connection of the segments with the fixed platform are:

 $A_1 = [690, 0, 0],$ $A_2 = [-345, 597.55, 0]$ et $A_3 = [-345, -597.55, 0]$

The points of connection of the segments with the mobile platform are:

 $\begin{array}{l} B_1 = [350, \, 0, \, 0], \\ B_2 = [-175, \, 303.1, \, 0] \text{ and } \\ B_3 = [-175, \, -303.1, \, 0] \end{array}$

In order to verify the obtained relations of both direct and inverse kinematics; we have performed a simulation by implementing the code corresponding to these relations. We provide the coordinates of some points in the attainable workspace of our robot as inputs and we check the results which are articular coordinates. Conversely, we introduce these results (articular coordinates) as inputs and we except to obtain the operational coordinates.

The experimental results reveal that the robot moves towards the vicinity of the position specified in operational co-ordinates by the user.

In addition, experiments of tele-operation were carried out on site via Intranet where the vision is ensured by a webcam. In these experiments, the operator controls the robot by giving commands (impulses) via a graphical user interface.

CONCLUSION

We have launched the analysis of a constrained parallel robot with three dof and four segments. We have chosen a particular structure of the passive segment to give a pure translation movement to the robot. We have obtained analytical solutions for the direct and inverse geometrical model. We have also obtained the Jacobian matrix by simple derivation.

Our theoretical expressions concerning inverse and direct kinematics have been well verified thought simulation. Nevertheless, in the practice, our results agree qualitatively with our theoretical expressions.

Indeed, errors are stemming from the fact that our model does not take into account the inertia of the system and the modeling of motors. In the same way, the absence of feedback control in this experimental prototype does not enable to compensate these errors. To improve the precision of our robot, the kinematics model and the feedback control are in the course of implementation.

REFERENCES

- V. E Gough. "Contribution to discussion of papers on research in automobile stability, control and tyre performance". Proceedings Auto Div. Inst. Mech. Eng. 1956-1957.
- [2] D. Stewart. "A platform with six degrees of freedom". UK Institution of Mechanical Engineers Proceedings. Vol. 180 N° 15, pp. 71-379. 1965-66.
- [3] K.H. Hunt and E.J.F. Primrose. "Assembly Configurations of some In-Parallel-Actuated Manipulators". Mechanism and Machine Theory. Vol. 28 N° 1, pp. 31-42. 1993.
- [4] J.P. Merlet. "Les robots parallèles". pp. 27-28 Hermes. Paris 1997.
- [5] J.P. Merlet and C. Gosselin. "Nouvelle architecture pour un manipulateur parallèle à six degrés de liberté". Mech. Mach. Theory. Vol. 26 N° 1, pp. 77-99. 1991.
- [6] L.W. Tsai. "Kinematics of three d.o.f. platform with three extensible limbs". Recent advances in robot kinematics, pp. 401-410. 1996.
- [7] D. Kim and W. K. Chung. "Kinematics condition analysis of three dof pure translational parallel manipulators". Journal of mechanical design. Vol. 125, pp. 323-331. 2003.
- [8] H. S. Kim and L.W. Tsai. "Kinematics synthesis of spatial 3- RPS parallel manipulator". Vol. 125, pp. 92- 97. 2003.
- [9] M. Carricato and V.P. Castelli. "A family of 3 d.o.f. translational parallel manipulators". Transactions of the ASME. Vol. 125, pp. 302-306. 2003.
- [10] M. Carricato and V.P. Castelli. "Position analysis of a new family of 3 d.o.f. translational parallel manipulators". Transactions of the ASME. Vol. 125, pp. 316-322.2003.
- [11] M. Stock and K. Miller. "Optimal Kinematics design of spatial parallel manipulators: application

of linear Delta-Robot". Transactions of the ASME. Vol. 125, pp. 292-300. 2003.

- [12] M. Callegari and M. Tarantini. "Kinematic analysis of translational platform". Translations of the ASME. Vol. 125, pp. 308-315. 2003.
- S.A. Joshi and L.W. Tsai. "The kinematics of a class of 3 DOF, 4-legged parallel manipulators". Journal of Mechanical Design. Vol. 125, pp. 52-60. 2003.
- [14] F.A. Adkins and E.J. Haug. "Operational envelope of a spatial Stewart platform". ASME J. of Mechanical Design. Vol. 119, pp. 330-332. 1997.
- J.R. Pennock and D.J. Kassner. "The work space of general geometry planar three-degree-offreedom plat-form type manipulator". ASME J. of Mechanical Design. Vol. 115, pp. 269-276. 1993.
- [16] R. Benea. "Contribution à l'étude des robots pleinement parallèles de type 6 R-RR-S". Thèse présentée à l'Université de Savoie. 1996.
- [17] M. Callegari and M. Tarantini. "Kinematic analysis of a novel platform". ASME J. of Mechanical Design. Vol. 125, pp. 308-314. 2003.
- [18] C. Gosselin. "Determination of the workspace of 6-dof parallel manipulators". ASME J. of Mechanical Design. Vol. 112, pp. 331-336. 1990.
- [19] E.J. Haug, F.A. Adkins and C.M. Luh. "Operational envelopes for working bodies of mechanisms and manipulators". Journal of mechanical design. ISSN 1050-0472. Vol. 120 N° 1, pp. 84-91. 1998.
- [20] A. Bonev and L. Ryu. "Workspace analysis of PRRS parallel manipulators based on the vertex space concept". Proceedings of DETC 99. September 12-15. Las Vegas. 1999.

- [21] I.A. Bonev and C.M. Gosselin. "A geometrical algorithm for the computation of the costantorientation workspace of 6-RUS parallel manipulators". ASME Design Engineering Technical Conferences (DETC 2000). Baltimore, MD, USA. September 10-13, 2000.
- [22] A. Cherfia, A. Zaatri and M. Giordano. "Kinematics analysis of a parallel robot with 3 DOF and 4 segments in pure translation". Mansoura Engineering journal (MEJ). Vol. 31 N° 2. 2006.
- [23] X.J. Liu. "Mechanical and kinematics design of parallel robotic mechanisms with less than six degrees of freedom". Postdoctoral Research Report Tsinghua University. Beijing, China. 2001.
- [24] X.J. Liu, J.I. Jeong and J. Kim. "A three translational DoFs parallel cube-manipulateur". Robotica. Vol. 21, pp. 645-653. Cambridge University Press. 2003.
- [25] A. Cherfia, A. Zaatri, M. Giordano and T. Seghiri.
 "Etude cinématique d'un robot parallèle à trois segments actifs et un segment central passif".
 4éme journées d'études techniques (JET'2006).
 Marrakech (Maroc) du 26 au 28 avril 2006.
- [26] J.P. Merlet. "Determination of 6d workspaces of Gough-type parallel manipulator and comparison between different geometries". Int. J. of Robotics Research. Vol. 18, pp. 902-916. 1999.
- [27] J.P. Merlet and C. Gosselin. "Nouvelle architecture pour un manipulateur parallèle à six degrés de liberté". Mech. Mach. Theory. Vol. 26 N° 1, pp. 77-99. 1999.
- [28] A. Cherfia, A. Zaatri, H. Heguib and S. Teyar. "Espace de travail et singularités d'un robot parallèle contraint à 3 ddl à translation pure". Conférence sur les sciences de la mécanique, oum el Bouaghi, 19-21. Novembre 2006.