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EINSTEIN EQUATIONS FOR TETRAD FIELDS

ECUACIONES DE EINSTEIN PARA CAMPOS TETRADOS

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RESUMEN

Todo tensor métrico puede ser expresado por el producto interno de campos tetrados. Se prueba que las ecuaciones de Einstein para esos campos tienen la misma forma que el tensor electromagnético de momento-energía si la corriente externa total es igual a cero. Usando la teoría de campo unificado de Evans se muestra que la verdadera unificación de la gravedad y el electromagnetismo es con las ecuaciones de Maxwell sin fuentes.

Palabras clave: Ecuaciones de Einstein, campos tetrados, tensor de momento-energía, geometría Riemann-Cartan, sistemas Einstein-Maxwell.

ABSTRACT

Every metric tensor can be expressed by the inner product of tetrad fields. We prove that Einstein's equations for these fields have the same form as the stress-energy tensor of electromagnetism if the total external current $j_\alpha = 0$. *Using the Evans' unified field theory, we show that the true unification of gravity and electromagnetism is with source-free Maxwell equations*

Keywords: Einstein equations, tetrad fields, metric tensor, energy tensor, electromagnetism, Riemann-Cartan geometry; Einstein–Maxwell system.

INTRODUCTION

It is agreed that gravitation can be best described by general relativity and that it cannot be explained by using fields as in electromagnetism as in the case of any other interaction. Furthermore, it has been assumed that the metric tensor is the best mathematical argument to use to study on gravitation. Such opinions led physicists to concentrate more on only the metric tensor and, hence, to change it according to circumstances. As a result, this method provides some important results about gravitation. However, it is also obvious that these results are not enough to understand gravitation as well as, perhaps, other interactions.

In the present paper, instead of concentrating on the metric tensor, we shall focus on tetrad fields. Our first objective will be to find some reasonable mathematical results with these fields. The complete interpretation of the results will be out of the scope of this paper.

Gravitation curves the space-time and this effect is related to the line element or invariant interval as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

where $g_{\mu\nu}$ is the metric tensor and its elements are some functions of the space-time.

The metric tensor with tetrad fields is given by [1, 2]

$$g_{\mu\nu} = e_\mu \bullet e_\nu \tag{1}$$

where e_μ are basis vectors or tetrad fields, and these are some functions of the space-time also (μ, $\nu = 0, 1, 2, 3$).

Similar to (1), the inverse metric tensor can be written as

$$g^{\mu\nu} = e^\mu \bullet e^\nu$$

where e^μ are basis vectors of the dual space or cotetrad fields. However, we will refer to these fields as inverse fields throughout this work.

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There are some useful features of and equations for the tetrad fields and inverse fields. First

$$
\begin{aligned}
&g^{\mu\alpha} g_{\alpha\nu} = \delta^{\mu}{}_{\nu} \\
&\mathrm{e}^{\mu} \bullet \mathrm{e}^{\alpha} g_{\alpha\nu} = \delta^{\mu}{}_{\nu} \\
&\mathrm{e}^{\mu} \bullet \mathrm{e}_{\nu} = \delta^{\mu}{}_{\nu}
\end{aligned}
\qquad (2)
$$

Other equations and all detailed calculations are given in the appendix section.

If the metric tensor is determined, it is well-known that it is demanding work to find the Einstein equations. The Christoffel symbols for the metric tensor (1) are

$$\Gamma^{\alpha}{}_{\mu\nu} = \frac{1}{2}\mathrm{f}^{\alpha}{}_{\nu} \bullet \mathrm{e}_{\mu} = \frac{1}{2}\mathrm{f}^{\alpha}{}_{\mu} \bullet \mathrm{e}_{\nu}$$

where $\mathrm{f}^{\alpha}{}_{\nu} = \partial^{\alpha}\mathrm{e}_{\nu} - \partial_{\nu}\mathrm{e}^{\alpha}$.

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The Riemann tensor for the above Christoffel symbols is

$$R^{\alpha}{}_{\mu\beta\nu} = \frac{1}{2}\partial^{\alpha}\mathrm{f}_{\beta\nu} \bullet \mathrm{e}_{\mu} + \frac{1}{4}\mathrm{f}^{\alpha}{}_{\nu} \bullet \mathrm{f}_{\beta\mu} + \frac{1}{4}\mathrm{f}^{\alpha}{}_{\beta} \bullet \mathrm{f}_{\mu\nu},$$

the Ricci tensor is

$$R_{\mu\nu} = \mathrm{j}_{\nu} \bullet \mathrm{e}_{\mu} + \frac{1}{4}\mathrm{f}^{\alpha}{}_{\nu} \bullet \mathrm{f}_{\alpha\mu}$$

and the Ricci scalar is

$$R = \mathrm{j}_{\beta} \bullet \mathrm{e}^{\beta} + \frac{1}{8}\mathrm{f}^{\alpha\beta} \bullet \mathrm{f}_{\alpha\beta}$$

where $\mathrm{j}_{\nu} = \frac{1}{2}\partial^{\alpha}\mathrm{f}_{\alpha\nu} = \partial^{\alpha}\partial_{\alpha}\mathrm{e}_{\nu}$ is the non homogeneous Maxwell equation.

Finally the Einstein Tensor can be expressed as

$$G_{\mu\nu} = \frac{1}{4}\left[\mathrm{f}^{\alpha}{}_{\nu} \bullet \mathrm{f}_{\alpha\mu} - g_{\mu\nu}\left(\frac{1}{4}\mathrm{f}^{\alpha\beta} \bullet \mathrm{f}_{\alpha\beta} + \mathrm{j}_{\alpha} \bullet \mathrm{e}^{\alpha}\right)\right]. \qquad (3)$$

The expression in square brackets is the same as the stress-energy tensor of electromagnetism except for the inner products. Despite this difference, the equations of motion of the tetrad fields have the same form as the Maxwell equations; that is $\partial^{\alpha}\partial_{\alpha}e_{\nu} = j_{\nu}$, with $j_{\alpha} = 0$ and is the Maxwell electromagnetic tensor

$$G_{\mu\nu} = \frac{1}{4}\left[F^{\alpha}{}_{\nu} \bullet F_{\alpha\mu} - g_{\mu\nu}\left(\frac{1}{4}F^{\alpha\beta} \bullet F_{\alpha\beta}\right)\right] \qquad (4)$$

Several results can be obtained from (3). However, the most significant of these is that the Einstein equations for the tetrad fields certainly give the electromagnetic stress-energy tensor. More precisely, the general relativity reveals that there are some inherent constraints for tetrad fields. This means there are some definite limits for the metric tensor. Since every metric tensor can be written in terms of tetrad fields, metric tensors cannot be chosen or adjusted arbitrarily. Instead, metric tensors must be found as inner products of tetrad fields after these fields are determined to be consistent with

$$\partial^{\alpha}\partial_{\alpha}\mathrm{e}_{\nu} = \mathrm{j}_{\nu} = 0.$$

Another formalism to obtain this result is with the unified field theory of Evans [3, 4]. We take the notation and the conventions from [1], where also more references to Evans' work can be found. We assume that the reader is familiar with the main content of tetrad formalism. Here we were able to reduce Evans' theory to just nine equations, which we will list again for convenience. Spacetime obeys in Evans' theory a Riemann-Cartan geometry (RC-geometry) that can be described by an orthonormal coframe e^{α}, a metric $g_{\alpha\beta}$ = diag (+1,–1,–1,–1), and a Lorentz connection $\Gamma^{\alpha\beta} = \Gamma^{\beta\alpha}$. In terms of these quantities, we can define torsion and curvature, respectively:

$$T^{\alpha} := De^{\alpha}, \qquad (5)$$

$$R_{\alpha}{}^{\beta} := d\Gamma_{\alpha}{}^{\beta} - \Gamma_{\alpha}{}^{\gamma} \wedge \Gamma_{\gamma}{}^{\beta}. \qquad (6)$$

The Bianchi identities and their contractions follow there from.

The extended homogeneous and inhomogeneous Maxwell equations read in Lorentz covariant form

$$
\begin{aligned}
&D\mathcal{F}^{\alpha} = R_{\beta}{}^{\alpha} \wedge \mathcal{A}^{\beta} \\
&D{*}\mathcal{F}^{\alpha} = {*}R_{\beta}{}^{\alpha} \wedge \mathcal{A}^{\beta},
\end{aligned}
\qquad (7)
$$

respectively. Alternatively, with Lorentz non-covariant sources and with partial substitution of (7), they can be rewritten as

$$d\mathcal{F}^{\alpha} = \kappa_0\left(R_{\beta}{}^{\alpha} \wedge e^{\beta} - \Gamma_{\beta}{}^{\alpha} \wedge T^{\beta}\right), \qquad (8)$$

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$$d{*}\mathcal{F}^{\alpha} = \kappa_0 \left({*}R_{\beta}{}^{\alpha} \wedge e^{\beta} - \Gamma_{\beta}{}^{\alpha} \wedge {*}T^{\beta} \right). \tag{9}$$

In the gravitational sector of Evans' theory, the Einstein-Cartan theory of gravity (EC-theory) was adopted by Evans. Thus, the field equations are those of Sciama [5], which were discovered in 1961:

$$\frac{1}{2}\eta_{\alpha\beta\gamma} \wedge R^{\beta\gamma} = \kappa\,\Sigma_{\alpha} = \kappa\left(\Sigma_{\alpha}^{\mathbf{mat}} + \Sigma_{\alpha}^{\mathbf{elmg}}\right), \tag{10}$$

$$\frac{1}{2}\eta_{\alpha\beta\gamma} \wedge T^{\gamma} = \kappa\,\tau_{\alpha\beta} = \kappa\left(\tau_{\alpha\beta}^{\mathrm{mat}} + \tau_{\alpha\beta}^{\mathrm{elmg}}\right). \tag{11}$$

Here $\eta_{\alpha\beta\gamma} = {*}\left(e_{\alpha} \wedge e_{\beta} \wedge e_{\gamma}\right)$. The total energy-momentum of matter plus electromagnetic field is denoted by Σ_{α}, the corresponding total spin by $\tau_{\alpha\beta}$.

What we will do here is to set a new principle where $\tau_{\alpha\beta}^{mat} + \tau_{\alpha\beta}^{elmg} = 0$, so that describes the truly unification of electromagnetism and gravitation. The derivation of the field equations and their properties are discussed in [7].

Now we have conditions to discuss the Unification of Electromagnetism and Gravitation through "Generalized Einstein tetrads" who H. Akbar-Zadeh has proposed [6] a new geometric formulation of Einstein–Maxwell system with source in terms of what are called "Generalized Einstein manifolds". We show that, contrary to the claim, Maxwell equations have not been derived in this formulation and that, the assumed equations can be identified only as source free Maxwell equations in the proposed geometric set up. A genuine derivation of source-free Maxwell equations is presented within the same framework. We draw a conclusion that the proposed unification scheme can pertain only to source-free situations.

In a recent article [6], using the tangent bundle approach to Finsler Geometry, H. Akbar-Zadeh has introduced a class of Finslerian manifolds called "Generalized Einstein manifolds'. These manifolds are obtained through some constrained metric variations on an action functional depending on the curvature tensors. The author has then proposed a new scheme for the unification of electromagnetism and gravitation, in which the spacetime manifold, M, with its usual pseudo-Riemannian metric, $g_{\mu\nu}(x)$, is endowed with a Finslerian connection containing the Maxwell tensor, $F^{\mu\nu}(x)$. Following this scheme, the author arrives at a class of Generalized Einstein manifolds containing the solutions of Einstein–Maxwell equations.

As for Maxwell equations, they are declared [1] to have been obtained by means of Bianchi identities. We wish to point out the following flaws in the treatment of Einstein-Maxwell system.

First consider the treatment of Maxwell equations. Through some constrained metric variations, and the use of Bianchi identities, the author arrives at [1, eq (5.55)]:

$$\nabla_{\mu} F^{\mu\nu} = \mu_1 u^{\nu}, \tag{12}$$

where μ_1 and $\mu^{\nu} = j^{\nu}$ are defined by [1, eqs (5.14) and (2.7)]: using notations of [3]

$$\mu_1 = -u^r \nabla_i F^i_r, \; u^r = \frac{v^r}{F}. \tag{13}$$

Using notations of [1] throughout, v^r are fiber coordinates of the tangent bundle over M and ∇_i is the usual Riemannian covariant derivative defined through $g_{ij}(x)$. Assuming that μ_1 is the proper charge density [1], the author then identifies (1) as the Maxwell equations with source. The author has, therefore, assumed that:

$$\mu_1 = \mu_1(x) \tag{14}$$

However, this assumption, together with definition (13), already implies equation (12). To see this, differentiate (13) with respect to v^j and then use (12) to obtain:

$$\nabla_i F_j^{\ j} = u^r \frac{\partial F}{\partial v^j} \nabla_i F_r^{\ i}$$

noting that $\frac{\partial F}{\partial v^j} = u_j$, and using (13) again, we arrive at (12). Therefore, rather than being derived, (1) has in fact been merely assumed.

More importantly, assumption (12) implies that $\mu_1 = 0$, so that the assumed equations can be identified only as source-free Maxwell equations. However, for a system of charged particles, for which we can write Maxwell equations, the velocity vector is a function of x. Therefore (12) can not be identified as Maxwell equations with source because μ^j in this equation are independent of x and (contrary to [6]) cannot be considered as a velocity field. There is, in fact, a genuine derivation of source-free. Consequently the proposed geometric formulation of Einstein–Maxwell system can pertain only to source-free situations. However if we include chiral currents (appendix 1) the truly unification of electromagnetism and gravitation is obtained [7].

CONCLUSION

We have shown that every metric tensor can be expressed by the inner product of tetrad fields. We have proved that Einstein equations for these fields have the same form as the stress-energy tensor of electromagnetism if the total external current $j_\alpha = 0$. Besides, using the unified field theory of Evans we show that the truly unification of gravity and electromagnetism is with the source free Maxwell equations. However a truly unification of electromagnetism and gravitation is obtained if chiral currents are included.

APPENDIX 1

In his 1916 paper on *The Foundation of the General Theory of Relativity* [8], Albert Einstein demonstrates the conservation of energy by relating the total energy tensor $T^\mu{}_\nu$ to the Bianchi identity $\left(R^\mu{}_\nu - \frac{1}{2}\delta^\mu{}_\nu R\right)_{;\mu} = 0$, the Maxwell energy tensor $T^\mu{}_{\nu\ Maxwell}$, the field strength tensor $F^{\mu\nu}$, and the energy tensor $t^\mu{}_\nu$ of the gravitational field are related according to:

$$
\begin{aligned}
-\kappa T^\mu{}_{\nu;\mu} &= -\kappa\left(T^\mu{}_{\nu\ Maxwell} + t^\mu{}_\nu\right)_{;\mu} \\
&= \left(R^\mu{}_\nu - \frac{1}{2}\delta^\mu{}_\nu R\right)_{;\mu} = 0 \\
&= \kappa\left[\frac{1}{2}F^{u\sigma}\left(F_{\mu\nu;\sigma} + F_{\nu\sigma;\mu} + F_{\sigma\mu;\nu}\right)\right] \\
&= \kappa\left[\left(F^{u\sigma}F_{\nu\sigma} - \frac{1}{4}\delta^\mu{}_\nu F^{\tau\sigma}F_{\tau\sigma}\right)_{;\mu} - F_{\nu\mu}F^{\sigma\mu}{}_{;\sigma}\right] \\
&= \kappa\left[\frac{1}{2}\left(F^{u\sigma}F_{\nu\sigma} + {*F^{u\sigma}} * F_{\nu\sigma}\right)_{;\mu} - F_{\nu\mu}F^{\sigma\mu}{}_{;\sigma}\right] = 0
\end{aligned}
\qquad (1.1)
$$

The "dual" of the field strength tensor above is defined as $*F^{\sigma\tau} \equiv \frac{1}{2!}\varepsilon^{\delta\gamma\sigma\tau}F_{\delta\gamma}$ using the Levi-Civita formalism, see, for example, [9, 10 and 12]. This also employs $\varepsilon^{\delta\mu\sigma}\varepsilon_{\alpha\beta\nu\sigma} = -\delta^{\delta\mu\sigma}{}_{\alpha\beta\nu\sigma}$, see [11-12]. Integral to the identity of $T^\mu{}_{\nu;\mu}$ with zero and thus to energy conservation is the second of Maxwell's equations:

$$\frac{1}{4}\left(F_{\tau\sigma;\nu} + F_{\sigma\nu;\tau} + F_{\nu\tau;\sigma}\right) = 0 \qquad (1.2)$$

which in turn has its identity to zero ensured by the Abelian relationship:

$$F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu} \qquad (1.3)$$

between the four-vector potential A^u and F^{uv}. Absent (1.3) above, or, if (1.3) above were to instead be replaced by the non-Abelian (Yang-Mills) relationship of the general form:

$$F_{i\mu\nu} = A_{i\nu;\mu} - A_{i\mu;\nu} - gf_{ijk}A^j{}_\mu A^k{}_\nu\,, \qquad (1.4)$$

where i is an internal symmetry index, f_{ijk} are group structure constants, and g is an interaction charge, then (1.2) would no longer be assured to vanish identically, and so the total energy tensor as specified in (1.1) would no longer be assured to be conserved, $T^\mu{}_{\nu;\mu} \neq 0$. More to the point, the total energy $T^\mu{}_\nu$ would no longer be "conserved", but would need to be exchanged with additional energy terms not appearing in (1.1). It is to be observed that non-linear $A{\cdot}A$ interaction terms such as in (1.4) are also central to modern particle physics, and so must eventually be accommodated by an equation of the form (1.1) if we are ever to understand weak and strong quantum interactions in a gravitational, geometrodynamic framework.

The set of connections in (1.1) do, of course, underlie the successful identification of the Maxwell – Poynting tensor for "matter" with the integrable terms in (1.1), according to:

$$T^\mu{}_{\nu\ Maxwell} \equiv -\left[F^{u\sigma}F_{\nu\sigma} - \frac{1}{4}\delta^\mu{}_\nu F^{\tau\sigma}F_{\tau\sigma}\right] = -\frac{1}{2}\left[F^{u\sigma}F_{\nu\sigma} + {*F^{u\sigma}} * F_{\nu\sigma}\right]1 \qquad (1.5)$$

as well as the identification of the non-integrable energy tensor $t^\mu{}_\nu$ of the "gravitational field":

$$\kappa_\nu \equiv t^\mu{}_{\nu;\mu} = F_{\mu\nu}F^{\sigma\mu}{}_{;\sigma} = F_{\mu\nu}J^\mu\,, \qquad (1.6)$$

which represents the density of energy-momentum exchanged per unit of time, between the electric current density J^μ and electromagnetic field $F_{\mu\nu}$ (see [12], following equation (65a)). In the above, we have employed Maxwell's remaining equation

$$J^{\nu} = F^{\mu\nu}{}_{;\mu} \tag{1.7}$$

However, if we set:

$$\begin{aligned} -\kappa T^{\mu}{}_{\nu\, Maxwell} &= R^{\mu}{}_{\nu} - \frac{1}{2}\delta^{\mu}{}_{\nu}R \\ &= \kappa\left(F^{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta^{\mu}{}_{\nu}F^{\tau\sigma}F_{\tau\sigma}\right) \\ &= \frac{\kappa}{2}\left(F^{\mu\sigma}F_{\nu\sigma} + {}^{*}F^{\mu\sigma}\,{}^{*}F_{\nu\sigma}\right) \end{aligned} \tag{1.8}$$

then, on account of (1.1), we find that $\kappa_{\nu} = 0$ in (1.6) and so the current is thought to vanish, $J^{\mu} = 0$. Additionally, the trace equation vanishes:

$$\begin{aligned} \kappa T_{Maxwell} = R &= -\kappa\left(F^{\mu\sigma}F_{\mu\sigma} - \frac{1}{4}\delta^{\mu}{}_{\mu}F^{\tau\sigma}F_{\tau\sigma}\right) \\ &= -\frac{\kappa}{2}\left(F^{\tau\sigma}F_{\tau\sigma} + {}^{*}F^{\tau\sigma}\,{}^{*}F_{\tau\sigma}\right) = 0 \end{aligned} \tag{1.9}$$

on account of the photon mediators of the electromagnetic interaction being massless, and therefore traveling [illegible] the speed of light. Thus, as stated by Einstein in 1919, "we cannot arrive at a theory of the electron [and matter generally] by restricting ourselves to the electromagnetic components of the Maxwell-Lorentz theory, [illegible] long been known" [13].

In addition to the problem of ma[illegible], there are other problems which arise from equation ([illegible]). Because (1.1) relies upon the Abelian field (1.3), it is simply not valid for non-Abelian fields. [illegible], without a reconsideration of (1.1), one cannot [illegible]ly the [illegible]al Theory of Relativity to non-Abelian interactions. This immediately bars understanding [illegible] weak interactions, or $SU(3)_{QCD}$ interactions, for example, in connection with Einstein's theory of gravitation.

Additionally, (1.1) excludes, *a priori*, the possibility that magnetic and electric current of electromagnetic nature might actually exist in nature. Here Einstein does not considerer chiral electric and magnetic currents. Our conjecture is that without particle current, $J^{\mu} = 0$, we can take into account chiral currents produced by the electromagnetic field, so we have $J^{\mu}_{chielectric} \equiv J^{\mu}_{(ce)} \neq 0$. Besides we considerer no magnetic monopoles but we include chiral magnetic currents, $J^{\mu}_{chimagnetic} \equiv J^{\mu}_{(cm)} \neq 0$ [15].

In particular, if we define the third-rank antisymmetric tensor (following and extending the Yablon' approach [12]):

$$J_{(cm)\tau\sigma\nu} \equiv \left(F_{\tau\sigma;\nu} + F_{\sigma\nu;\tau} + F_{\nu\tau;\sigma}\right), \tag{1.10}$$

and because the current four-vector for chiral magnetic currents may be specified in terms of $J_{(cm)\tau\nu\sigma}$ and $^{*}F^{\mu\nu}$ by

$$J^{\sigma}_{(cm)} = {}^{*}J^{\sigma}_{(cm)} = \frac{1}{3!}\varepsilon^{\alpha\tau\gamma\sigma}J_{(cm)\alpha\tau\gamma} = {}^{*}F^{\mu\nu}{}_{;\mu} \tag{1.11}$$

we see that (1.1), as it [illegible]ds, expressly *forecloses* the existence of magnetic monopoles and chiral magnetic currents, because the vanishing of $J_{(cm)\tau\sigma\nu}$ in (1.10) causes $J^{\sigma}_{(cm)}$ in (1.11) to vanish as well. Any theory which allows chiral currents by using a non-Abelian field (1.4), requires that (1.1) be suitably-modified for total energy to be properly conserved, because $\left(F_{\mu\nu;o} + F_{\nu\sigma;\mu} + F_{\sigma\mu;\nu}\right)$ will no longer be identical to zero. For completeness, we also define (see [5]):

$$J_{(ce)\tau\sigma\nu} \equiv -\left({}^{*}F_{\tau\sigma;\nu} + {}^{*}F_{\sigma\nu;\tau} + {}^{*}F_{\nu\tau;\sigma}\right) = \frac{1}{3!}\varepsilon_{\gamma\tau\sigma\nu}J^{\gamma}_{ce} \tag{1.12}$$

As we shall demonstrate, all of theses problems stem from the fact that (1.1) relies upon the vanishing of the antisymmetric combination of terms in (1.2) to enforce the conservation of total energy. The term $T^{\mu}{}_{\nu;\mu} = 0$ is solidly-grounded: it is the quintessential statement that total energy must be conserved. The Bianchi identity $\left(R^{\mu}{}_{\nu} - \frac{1}{2}\delta^{\mu}{}_{\nu}R\right)_{;\mu} = 0$ is equally solid: although one can also add a "cosmological" term $\left(R^{\mu}{}_{\nu} - \frac{1}{2}\delta^{\mu}{}_{\nu}R + \Lambda\delta^{\mu}{}_{\nu}\right)_{;\mu} = 0$, one is assured by the very nature of Riemannian geometry that either combination of terms will always be zero. Not so, however, for $\frac{1}{2}F^{\mu\sigma}\left(F_{\mu\nu;o} + F_{\nu\sigma;\mu} + F_{\sigma\mu;\nu}\right) = 0$. This term relies directly on the Abelian field (1.3) and on the supposition that chiral magnetic currents (1.11) vanish. Absent this supposition, $T^{\mu}{}_{\nu}$ is no longer conserved, and so can no longer be regarded as the "total" energy tensor.

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