

Combinatorial optimization algorithm with coordinate search method to locate and size distributed generation plants in a radial distribution electrical system

Algoritmo de optimización combinatoria con método de búsqueda por coordenadas para ubicar y dimensionar centrales de generación distribuida en un sistema eléctrico de distribución radial

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ABSTRACT

This work developed a hybrid optimization algorithm called Combinatorial Optimization with a Coordinate Search Method, which allows for finding the optimal locations and capacities of distributed generation plants in an electrical distribution network. The algorithm was validated in the IEEE 69-bus test system, compared with the Elephant Herding Optimization method. Three scenarios were defined that were executed on the test system. In the first scenario, up to four distributed generation plants were located, reducing the value of the objective function to 0.0177. In the second scenario, satisfactory solutions were found that satisfied critical voltage and current restraints in the electrical system. All stresses were between 1.00 and 1.05 per unit for the first case. In the second case, two lines had the maximum current to flow through them limited to 1.00 ampere. In the third scenario, the algorithm was executed in the presence of capacitor banks. In this case, satisfactory results were also obtained, with reagent consumption in one of the distributed generation plants.

Keywords: Distributed generation, multi-objective optimization, loss reduction, voltage profile and load flow.

RESUMEN

En este trabajo se desarrolló un algoritmo de optimización híbrido denominado Optimización Combinatoria con Método de Búsqueda por Coordenadas, el cual permite encontrar las ubicaciones y capacidades óptimas de centrales de generación distribuida en una red de distribución eléctrica. El algoritmo fue validado en el sistema de prueba del IEEE de 69 nodos, siendo comparado con el método llamado Elephant Herding Optimization. Se definieron tres escenarios que se ejecutaron sobre el sistema de prueba. En el primer escenario se ubicaron hasta cuatro centrales de generación distribuida, reduciendo el valor de la función objetivo a 0,0177. En el segundo escenario se encontraron soluciones satisfactorias que cumplían las restricciones críticas de tensión y corriente en el sistema eléctrico. Para el primer caso, todas las tensiones estaban entre 1 y 1,05 por unidad. En el segundo caso, dos líneas tenían restricciones de corriente máxima fijadas en 1 ampere. En el tercer escenario el algoritmo se ejecutó en presencia de bancos de condensadores. En este caso también se obtuvieron resultados satisfactorios, con consumo de reactivo en una de las centrales de generación distribuida.

Palabras claves: Generación distribuida, optimización multiobjetivo, reducción de pérdidas, perfil de tensión y flujo de cargas.

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INTRODUCTION

Distributed generation (DG) is connected at points close to the consumers of an electrical distribution system [1]-[3]. The use of non-renewable energy sources has caused an increase in temperatures in recent years, and an increase in global warming has been observed [4]. Numerous efforts are being made to reduce dependence on fossil fuels in electricity generation today. Renewable energy sources are a viable alternative to achieve this, and DG offers an attractive way to integrate increasing volumes of these sources into the global energy matrix. In the future, the decentralized generation system will predominate in producing electricity globally. Today's increase in the use of microgrids and DG points for this to happen [5]. For example, by the year 2050, it is estimated that the presence of renewable energy sources in the world energy matrix will be 28% [6].

In Cuba, by 2030, electricity consumption is expected to increase by approximately 40%. In response to this growth, the policies continue to increase the generation matrix based on integrating renewable energy sources [7]. This increase in generation through renewable energy sources is projected to be no less than 24%, representing a considerable increase compared to the 4.6% in 2013 [8]. This increase in renewables has its legal protection in Decree-Law No. 345, where using this type of energy source is established as one of the priorities [9]; therefore, integrating this type of generation is safe, especially in distribution networks.

Optimally locating and sizing distributed generation plants (DGP) allows for reducing active power losses, improving the voltage profile, chargeability, reliability, and stability of an electrical distribution system. Additionally, it improves the quality of the energy delivered to consumers and the power factor of the network. The methods to solve this task can be analytical, employing classical optimization techniques, artificial intelligence, or hybrid techniques [10].

Some authors have developed analytical techniques that minimize the active power losses of a distribution system through the installation of distributed generation. These methods use sensitivity indices that indicate the change in system losses concerning

the current injection of a DGP, being able to find the best location, point of operation, and capacity [11]. There are other analytical methods, such as the Novel method (simple and with modification), combined lost sensitivity, index vector, and voltage sensitivity index [12].

Among the classic optimization techniques is dynamic programming (DP), not only to reduce losses but also to improve system reliability. Another classic technique combines nonlinear programming (NLP) and mixed integer programming (MIP) [13]. Another classic method used to reduce losses in a distribution system is the Mixed Integer Nonlinear Programming method, which consists of two steps and reduces the search space and calculation time [14].

Artificial intelligence-based techniques include genetic algorithm (GA) and particle swarm optimization (PSO). Some authors use these techniques to optimize the distribution plan of generators that use renewable sources as primary energy or to reduce losses and improve the voltage profile of distribution systems [15]-[16]. Another artificial intelligence technique is known as Gray Wolf Optimization (GWO). This AI has been used to size and locate small photovoltaic plants and wind turbines to improve the voltage profiles of electrical distribution networks [17].

PSO is used with Monte Carlo simulations to form a hybrid technique to solve the optimization problem better. Improved versions have been presented in this line, where strategies are used to update the position of the particles. The selection of the strategies is determined by a dynamic probability thanks to the generation of random numbers with the Monte Carlo method [18]. Backtracking Search Optimization Algorithm (BSOA) is an evolutionary method that consists of five steps for its execution. The novelty of this technique is that it uses two new operators to perform the crossover and mutation. BSOA provides a rapid response in finding adequate solutions for non-linear and non-differential optimization problems with complex optimization functions [19].

There is a balance between execution time and accuracy in method results since both are important to ensure the efficiency of an optimization algorithm. The trend is to develop algorithms that use the generation of random particles in a multidimensional

search space that describes the distribution system where the DGP is to be installed.

This work aims to develop an optimization algorithm capable of finding a suitable solution to critical operational restraints of an electrical distribution system. The optimization method developed is a hybrid algorithm called Combinatorial Optimization with Coordinate Search Method (COCSM).

METHODS

The COCSM algorithm uses a multi-objective function (F) that allows optimizing the location and dimensioning of m distributed generation plants in the m best buses of a radial electrical distribution network consisting of n buses, following three optimization criteria which are: minimize total active power losses, minimize system voltage deviation, or minimize both. Two optimization problems have to be solved jointly to achieve this. The first problem is based on finding the buses where the plants should be installed through an exhaustive search. The second problem is based on finding the plants' optimal capacities through a heuristic search algorithm. Should the user decides to analyze all the buses in the distribution network, the exhaustive search will have a total of $\binom{n}{m}$

possible solutions if no restraint is violated, where at least one of them will be the optimal. One of the advantages of this type of search method is that it allows finding a good solution whenever it exists, even if the restraints of the optimization problem are very tight. In other words, it favors the solution of the algorithm when the system is poorly conditioned. The optimal solution to the optimization problem will be comprise of a matrix (Π) of order m by n and a complex vector (\underline{C}) of order m . The matrix (Π) will contain the optimal location of the plants to be installed. Furthermore, the elements \underline{c}_j of the vector (\underline{C}) will be complex numbers that will store the optimal generation of active and reactive power of each plant connected to the distribution system.

Algorithm to generate each possible assignment of the optimization problem

The search space for the optimization problem is made up of $\binom{n}{m}$ possible solutions, where each solution i will have an associated assignment

$(\Pi)^i$. An assignment is no more than an arrangement or location of the plants m , in the n distribution system buses. Since the optimization algorithm allows dimensioning the capacity of the generation plants that will be located, it is optional to analyze the case where two or more plants are installed in the same bus. The elements π_{jk}^i of the matrices $(\Pi)^i$ must satisfy the following restriction:

$$\pi_{jk}^i = \{0,1\} \quad (1)$$

$$\sum_{j=1}^m \pi_{jk}^i = 1 \quad (2)$$

$$\sum_{k=1}^n \pi_{jk}^i = 1 \quad (3)$$

Where:

$$j = \overline{1,m}; k = \overline{1,n}$$

π_{jk}^i : Elements of the allocation matrix $(\Pi)^i$.

The above restrictions allow that each DGP can only be connected to one system bus and that only one plant can be installed in each bus. Next, each step of the algorithm that allows for generating the assignments $(\Pi)^i$ will be explained.

Step 1: For the algorithm to work, the first assignment matrix $(\Pi)^1$ of order m by n needs to be constructed. Figure 1 shows the structure of this matrix in which the m DGPs are assigned to the first m buses of the matrix. This procedure yields a matrix containing a 1 in the elements of the main diagonal of the first m -order square submatrix of the matrix $(\Pi)^i$. In other words, means that the first m buses of the electrical distribution system will be assigned one and only one plant.

Step 2: This step allows to form the new matrices that will be stored in memory. The objective is to move the DGP located in bus m to the next bus, exchanging its position in row m of the matrix (Π) to generate a new assignment i . Figure 2 shows this process in a more detailed way. When DGP "m" reaches bus "n," its position in the matrix $(\Pi)^i$ is changed so that it occupies column $m + 1$. Instead, the DGP of row $m - 1$ will now occupy column m . This process can be seen in Figure 3. Each plant will repeat this process, ensuring that no bus (columns) have two plants installed and that the entire search space is traversed.

$$(\Pi)^1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & \dots & m & m+1 & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 \\ & & \vdots & & \vdots & \\ 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix} \end{matrix}_{m \times n}$$

 Figure 1. First assignment matrix $(\Pi)^1$ of the optimization problem.

$$(\Pi)^i = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & \dots & m & m+1 & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 \\ & & \vdots & & \vdots & \\ 0 & 0 & 0 & \dots & 1 & \dots \end{pmatrix} \end{matrix}_{m \times n}$$

 Figure 2. Permutation of the distribution generation plant from row m to the buses that are after it in the system.

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & \dots & m & m+1 & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 \\ & & \vdots & & \vdots & \\ 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix} \end{matrix}_{m \times n} \xrightarrow{\quad} \begin{matrix} & \begin{matrix} 1 & 2 & 3 & \dots & m & m+1 & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 \\ & & \vdots & & \vdots & \\ 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix} \end{matrix}_{m \times n}$$

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 & \dots & m & m+1 & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 \\ & & \vdots & & \vdots & \\ 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix} \end{matrix}_{m \times n} \xrightarrow{\quad} \begin{matrix} & \begin{matrix} 1 & 2 & 3 & \dots & m & m+1 & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ & & \vdots & & \vdots & \\ 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix} \end{matrix}_{m \times n}$$

 Figure 3. Transformations of the matrix (Π) to modify the location of the plant that is in the position $j - 1$.

Stopping criterion This algorithm ends when all $\binom{n}{m}$ possible assignments are generated, and it is verified which of them do not violate any of the restraints of the optimization problem.

Coordinate search method

The Coordinate Search Method [20] (CSM) is based on performing one-dimensional searches in the directions of the objective function variables. Its use is effective if this function is continuous and differentiable in such a way that it is possible to obtain a solution if it exists.

The active and reactive powers generated by the DGPs that are installed in the buses of an assignment $(\Pi)^i$ are defined by the elements $p_{GD_j}^i$ and $q_{GD_j}^i$ of the vectors $(P_{GD})^i$ and $(Q_{GD})^i$ of order m . The values of these elements will be updated during each iteration t of the CSM, to achieve an optimal result for the

solution i . This result is a local optimum of the optimization problem since only the best capacities are found for the assignment $(\Pi)^i$. A complex vector $(\underline{C})^i$ stores each of the $\binom{n}{m}$ solutions to subsequently determine the best one, considering the value of the selected objective function.

The optimization problem will be assumed to have at least one solution to explain the CSM. Initially, two vectors $(P_{GD})^0$ and $(Q_{GD})^0$ must be found that will constitute the initial active and reactive power generation of each DGP, such that they guarantee that no restriction of the optimization problem is breached. If no distribution system restraint is violated when operating without the presence of distributed generation, all elements of the vectors $(P_{GD})^0$ and $(Q_{GD})^0$ will be 0. Otherwise, the system branches with the highest energy transfer are detected, and a plant is located downstream of it with a capacity

that can reduce the transfer of that line to 0. This strategy is only sometimes effective because it depends on the restraints of the plants; however, it almost always allows for finding good solutions, especially when system restraints are critical. After finding an initial solution, runs the CSM to look for a good solution for the capacities associated with the assignment $(\Pi)^i$.

In each iteration of the CSM, only one of the elements of the vectors $(P_{GD})^t$ and $(Q_{GD})^t$ will be increased or decreased with a value predetermined by the user and defined by the variable δ . The user will initially chose the variable and change its magnitude and sign so that the one-dimensional searches change direction and reduce their search step, ensuring higher resolution. In each iteration t of the CSM, the algorithm must perform τ searches since the searches try to optimize both the active and reactive power of the power plants, the variable $\tau = \overline{1, 2m}$. Two functions: $\Lambda_p(\tau)$ and $\Lambda_Q(\tau)$ and a vector $(L)^\tau$ of order m must be defined, which allow a one-dimensional search to be carried out at the same time. The functions $\Lambda_p(\tau)$ and $\Lambda_Q(\tau)$ will enable to perform searches that will optimize the active and reactive power of the power plant. These are defined in equation (4) and equation (5).

$$\Lambda_p(\tau) = \begin{cases} \delta & \tau = 1, 3, 5 \dots 2m-1 \\ 0 & \tau = 2, 4, 6 \dots 2m \end{cases} \quad (4)$$

$$\Lambda_Q(\tau) = \begin{cases} 0 & \tau = 1, 3, 5 \dots 2m-1 \\ \delta & \tau = 2, 4, 6 \dots 2m \end{cases} \quad (5)$$

The element l_j^τ of the vector $(L)^\tau$ allow only one of the plants to participate in the search. This element is subject to the constraint equation (6).

$$l_j^\tau = \{0, 1\} \quad (6)$$

The elements v_k^τ and w_k^τ represent the values of the active and reactive generation installed in each bus of the system, during a one-dimensional search. They compose the vectors $(V)^\tau$ and $(W)^\tau$ of order n . The values v_k^τ and w_k^τ will be equal 0 if no DGP is installed in a bus. These elements will also be updated during each iteration t of the method. The vectors $(P_{GD})^t$, $(Q_{GD})^t$, $(V)^\tau$ and $(W)^\tau$ are related to each other by equation (7) and equation (8).

$$(V)_n^\tau = [(P_{GD})_m^t + \Lambda_p(\tau) \cdot (L)_m^\tau] \cdot (\Pi)_{mxn}^i \quad (7)$$

$$(W)_n^\tau = [(Q_{GD})_m^t + \Lambda_Q(\tau) \cdot (L)_m^\tau] \cdot (\Pi)_{mxn}^i \quad (8)$$

The flow diagram for executing the one-dimensional searches of the CSM is shown in Figure 4.

First, there must be loaded the matrix $(\Pi)^i$ order m by n , the vectors $(P_{GD})^t$, and a vector $(L)^0$ of order m whose elements will be 0. The functions $\Lambda_p(\tau)$ and $\Lambda_Q(\tau)$ must also be loaded. Then the element l_j^τ at position $j + 1$ will receive the value 1. The vectors $(V)^\tau$ and $(W)^\tau$ will be stored in the elements of the vectors $(Y)^\tau$ and $(\Phi)^\tau$ at each iteration. Thereby, it is possible to store all one-dimensional searches that occur in one iteration of the CSM, and then choose the best one. The first conditional statement checks whether the variable τ took an even value. Thus, the vector $(L)^\tau$ remains unchanged until all

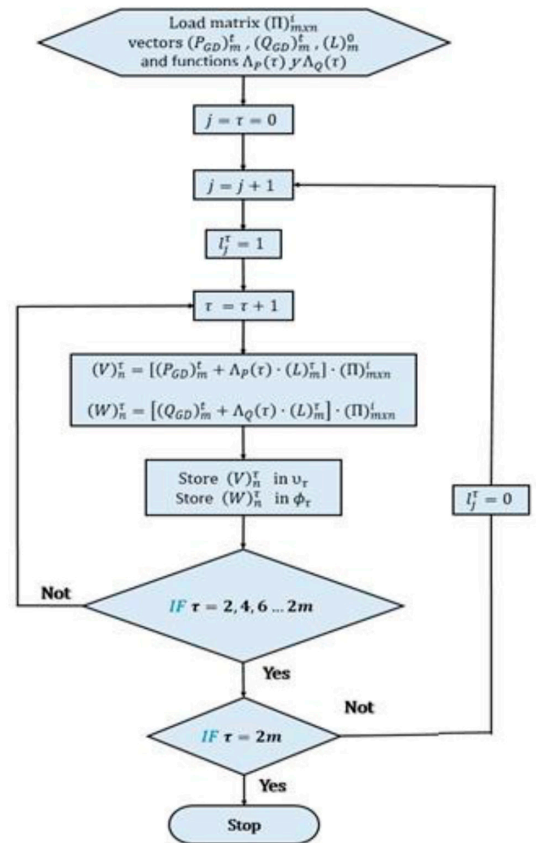


Figure 4. Flowchart of the coordinate search method.

one-dimensional searches for the same plant have been performed. If the value of τ is even, a check is made to see if all searches for iteration t of the CSM have been performed. If the statement is satisfied, the algorithm terminates. Otherwise, the value of element l_j^τ is changed to position $j + 1$. Figure 5 shows the transformations that the vector $(L)^\tau$ undergoes during the entire execution of the algorithm.

Haque method to solve load flows in radial networks

The algorithm employs a procedure known as power flow. This recursive method with non-linear equations allows knowing the state of the parameters that vary in the electrical network when modified due to the connection of one or more DGP. The method is used for the solution of the power flow in radial networks due to Haque [21].

The nominal voltage at which the electrical distribution system operates must first be known to execute the power flow. This value is considered the optimal level of voltage that each bus should have. The variable that will store this value is called U_{nom} . A vector $(\underline{U})^\tau$ of order n must also be created whose element \underline{u}_k^τ is a complex number that represents the bus voltage k for iteration τ . It is also necessary to define a vector $(U)^\tau$ with the modules of the voltages of each bus of the system. The vectors (P) and (Q) of order n , contain the elements p_k and q_k which are constant for each iteration and represent the active and reactive powers of the electrical loads installed in the bus k of the distribution system. The elements P_T^τ and Q_T^τ should also be defined to represent the active and reactive powers injected into the bus k . The injected power is nothing more than the difference between the demand and the generation existing in a bus when the system operates steadily. These elements conform the vectors $(P_T)^\tau$ and $(Q_T)^\tau$ of order $n-1$.

Vectors $(V)^\tau$ and $(W)^\tau$ are related to the vectors $(P_T)^\tau$ and $(Q_T)^\tau$ by equation (9) and equation (10), which

$$(L)^\tau = [1 \overset{1}{\curvearrowright} \overset{1}{\curvearrowright} \dots 0]_m$$

Figure 5. Transformations that the vector $(L)^\tau$ undergoes during the execution of the one-dimensional searches of the CSM.

allow calculating the powers injected in each bus to execute the power flow with the demand of the electrical system under the presence of distributed generation installed.

$$(P_T)^\tau = (P)_n - (V)_n^\tau \quad (9)$$

$$(Q_T)^\tau = (Q)_n - (W)_n^\tau \quad (10)$$

Defining the elements r_k and x_k of the vectors (R) and (X) of order $n-1$ is important. These represent the resistance and reactance value of the electrical distribution system line k . These elements are always kept constant regardless of the location of the DGP. Two vectors must be used to identify which buses are connected by each distribution system line. These vectors are (Ψ) and (Ω) of order $n-1$ and their elements Ψ_k and ω_k identify the sending and receiving buses of line k of the network. The sending bus is known as the bus where the feeder starts and the receiving one where it ends.

Another complex vector $(\Delta S)^\tau$ is created to store the results of active and reactive power losses in its elements Δs_k^τ . Equation (11) and equation (12) are used to calculate the active and reactive power losses for iteration τ in each line of the distribution system and the voltages of each bus.

$$\Delta s_k^\tau = \frac{[P_T^\tau]_k^2 + [q_T^\tau]_k^2}{[u_k^\tau]^2} \cdot (r_k + jx_k) \quad (11)$$

$$\underline{u}_{k+1}^\tau = \underline{u}_k^\tau - \frac{P_T^\tau - jq_T^\tau}{[u_k^\tau]^*} \cdot (r_k + jx_k) \quad (12)$$

Where:

$$k = \overline{1, n-1}$$

u_k^τ : Element k of vector $(U)^\tau$.

\underline{u}_k^τ : Element k of vector $(\underline{U})^\tau$. The element u_1 is always constant for each iteration.

The user establishes the load flow convergence through a tolerance, defined by the variable ϵ , which allows setting the stopping criterion (see equation 13). The power flow is an iterative method, so its convergence will occur when

the difference between the voltage of the same bus in the current and the previous iteration is very small.

$$\left| \underline{u}_{k_{actual}}^{\tau} - \underline{u}_{k_{anterior}}^{\tau} \right| < \varepsilon; \forall k = \overline{2, n} \quad (13)$$

Where:

$\underline{u}_{k_{anterior}}^{\tau}$: Voltage at the bus k for the iteration τ , in the previous iteration of the power flow.

$\underline{u}_{k_{actual}}^{\tau}$: Voltage at the bus k for the iteration τ , in the current iteration of the power flow.

Optimization problem restraints

Equality and inequality restraints must be established to avoid illogical results or results that do not meet the requirements of power system operation. Sometimes these restraints could prevent the global minimum of the objective function from being found; however, a good solution could be found by the algorithm guaranteeing a reliable operation of the radial distribution power system.

Equality restrictions

The equality restraints of the optimization problem are defined by equation (14) and equation (15).

$$P_G^{\tau} = P_D^{\tau} + \sum_{k=1}^{n-1} \text{real}(\Delta \underline{s}_k^{\tau}) \quad (14)$$

$$Q_G^{\tau} = Q_D^{\tau} + \sum_{k=1}^{n-1} \text{imag}(\Delta \underline{s}_k^{\tau}) \quad (15)$$

Where:

P_G^{τ} and Q_G^{τ} : they are the sum of the active and reactive powers delivered by all the generation plants and the electrical substation of the distribution system.

P_D^{τ} and Q_D^{τ} : they are the sum of the active and reactive powers consumed by all the electrical loads installed in the distribution system.

The previous equations guarantee the power balance of the system, fulfilling the energy conservation law.

Inequality restrictions

The inequality restraints of the optimization problem are presented in equations (16)-(20).

$$U_{\min} \leq u_k^{\tau} \leq U_{\max} \quad (16)$$

$$i_k^{\tau} \leq i_{k, \max} \quad (17)$$

$$(V_{\min})^{\tau} \leq (V)^{\tau} \leq (V_{\max})^{\tau} \quad (18)$$

$$(W_{\min})^{\tau} \leq (W)^{\tau} \leq (W_{\max})^{\tau} \quad (19)$$

$$\frac{v_k^{\tau}}{\sqrt{(v_k^{\tau})^2 + (w_k^{\tau})^2}} \geq fp \quad (20)$$

The restraint (16) allows the voltage profile of the distribution system to be between the minimum and maximum voltage values established by the user (U_{\min} and U_{\max}). The vector $(I)_{\tau}$ of order $n - 1$ stores the absolute values of the currents circulating each system feeder. The elements that make up this vector have the notation i_k^{τ} and are calculated by equation (21).

$$i_k^{\tau} = \frac{\left[P_T^{\tau} \psi_k \right]^2 + \left[q_T^{\tau} \psi_k \right]^2}{\left[u_{\psi_k}^{\tau} \right]^2} \quad (21)$$

Where:

$P_T^{\tau} \psi_k$ and $q_T^{\tau} \psi_k$: Active and reactive powers injected into the bus that occupies the position k of the vector (Ψ) .

The elements $i_{k, \max}$ of the vector (I_{\max}) of order $1n - 1$ represent the maximum current limits that can flow through the system feeders. It is worth clarifying that the elements of this vector are constant for each assignment of the search space. Equation (18) and equation (19) guarantees that the generation of active and reactive power assigned to each of the buses are bounded by the elements of the vectors $(W_{\min})^{\tau}$ and $(W_{\max})^{\tau}$ (for active power) and $(W_{\min})^{\tau}$ and $(W_{\max})^{\tau}$ (for reactive power). These vectors represent the minimum and maximum distributed generation capacities that can be installed in each bus of the electrical system. Equation (20) guarantees that the installed plants do not operate at a lower power factor than that defined by the constant fp .

Objective function of the algorithm

This work uses a multi-objective function (F) composed of two variables that depend on the power flow results. The first variable is associated with the system's active power losses (F_{AP}). Equation (22)

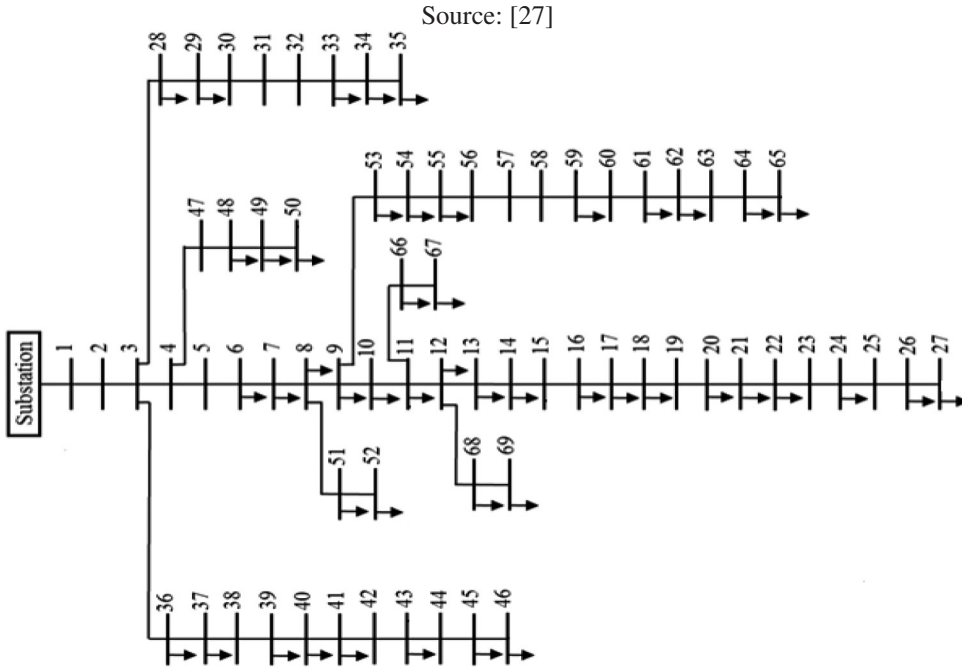


Figure 6. One-line diagram of the IEEE medium voltage 69-buses system.

allows these losses to be calculated by adding the real part of the apparent power loss vector.

$$F_{\Delta P} = \sum_{k=1}^{n-1} \text{real} (\Delta \underline{S}_k^{\tau}) \quad (22)$$

Where:

$\Delta \underline{S}_k^i$: Element k of complex vector $(\Delta \underline{S})^{\tau}$ of order $n - 1$ contains the values of the active and reactive power losses of each feeder of the electrical distribution system n : Buses of the radial distribution system.

The voltage profile can be improved by reducing the difference between the absolute value of each bus's voltage and the system's nominal voltage. One of the best ways to achieve this goal is to use the voltage mean square deviation (VMSD) as the adjustment parameter for the system voltages. Using the VMSD (variable $F_{\Delta U}$) makes it possible to reduce the deviations between the system and nominal voltages. The variable $F_{\Delta U}$ will depend on the sum of the square of the voltage deviations at each system bus and is defined by equation (23).

$$F_{\Delta U} = \frac{1}{n} \cdot \sum_{k=1}^n \left[1 - \frac{u_k^{\tau}}{U_{nom}} \right]^2 \quad (23)$$

Where:

$(u)_k^{\tau}$: Element k of vector $(U)^{\tau}$ of order n representing the voltage at each bus of the distribution system. These variables vary depending on the assignment being analyzed.

U_{nom} : Nominal voltage of the electrical system under study.

Formulation of the multiple objective function

The multi-objective function comprises the two variables $F_{\Delta P}$ and $F_{\Delta U}$, a weight factor called θ , and a penalty factor a . It is necessary to calculate the active power losses of the system (ΔP_0) and its VMSD (ΔU_0) in its initial state. These values are obtained using equation (22) and equation (23) after running a power flow without DGP. The expression that allows obtaining the value of F is defined by equation (24).

$$F \left[(\Pi)^i, (\underline{C})^i \right] = a \cdot \left[\theta \cdot \frac{F_{\Delta P}}{\Delta P_0} + (1 - \theta) \cdot \frac{F_{\Delta U}}{\Delta U_0} \right] \quad (24)$$

Where:

$$a = \begin{cases} D & \text{if (16) and (17) are not fulfilled} \\ 1 & \text{in any other case} \end{cases}$$

The variable D can take any value of the positive real numbers (\mathbb{R}^+) as long as it is large enough to prevent that solution from being considered. The variable a will only take the value of D if any system constraint is violated.

The user must chose the weight factor. The total of active power losses are minimized. If $\theta = 1$, but if $\theta = 0$, only the voltage profile will be improved. Assigning intermediate values to this weight factor will allow the objective function to reduce the system's VMSD and the active power losses.

Finally, the mathematical formulation of the optimization problem will be defined by:

$$\min_i F[(\Pi)^i, (\mathcal{C})^i]$$

Subject to restrictions equations (1)-(3), equation (6) and equations (14)-(20).

Description of the COCSM

The flow diagram of the Combinatorial Optimization with the Coordinate Search Method algorithm is shown in Figure 7. This diagram is developed in 5 steps that will be explained below.

Step 1: In this step, all the data of the electrical distribution system, the number of distributed generation plants to be installed, and the restraints of the optimization problem are entered. A power flow is also executed to calculate the initial value of active power losses and the initial value of VMSD. Then each of the assignments $(\Pi)^i$ are generated. Furthermore, it is important to define a constant ϵ that represents the minimum resolution for one-dimensional searches of the CSM. The CSM ends the search once the absolute value of the method's search step (represented by the variable δ) is less than the constant.

Step 2: The initial conditions to apply the CSM are established. First, find the two vectors $(P_{GD})^0$ and $(Q_{GD})^0$ that satisfy all the restrictions of the optimization problem in this step. Then a power flow is run to calculate the value of the objective function with these vectors, and they are set equal to the first elements of the vectors $(P_{GD})^\tau$ and $(Q_{GD})^\tau$ so that the CSM can be executed without errors.

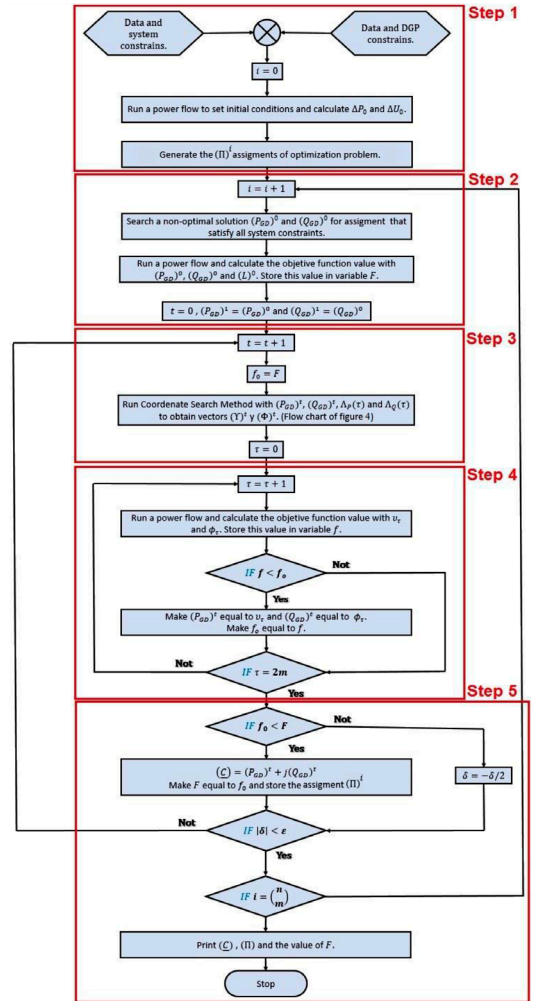


Figure 7. COCSM flow diagram.

Step 3: The coordinate search method is executed to obtain the vectors $(Y)^\tau$ and $(\Phi)^\tau$, which will store the vectors $(V)^\tau$ and $(W)^\tau$ that will be used to execute the power flow from step 4.

Step 4: The power flow executed in this step is used to evaluate the objective function for each one-dimensional search performed by the CSM. Each time the value of this function is less than the one previously saved, the vectors $(P_{GD})^\tau$ and $(Q_{GD})^\tau$ will be updated to obtain the solution vector (\mathcal{C}) in step 5.

Step 5: It is checked if the value of the objective function of the current iteration is less than the one that had been stored in past iterations. This result

is stored if this is so. The value of the analyzed assignment is stored in the solution matrix (Π) and the solution vector (\underline{C}) is formed by equation (25).

$$(\underline{C}) = (P_{GD})^t + j(Q_{GD})^t \quad (25)$$

When the conditional statement is not true, it means that none of the one-dimensional searches permitted to obtain a better result for the objective function compared to the results found in previous iterations of the CSM. In this case, the searches must be carried out in another direction with a short step. Changing the direction and the search step modifies the variable δ used by the $\Lambda_P(\tau)$ and $\Lambda_Q(\tau)$ functions.

Finally, it is checked if it is convenient to conclude the CSM for the possible solution i analyzed. If so, it is verified if all the possible locations each power plant can have in the electric distribution system have been analyzed. Finally, the algorithm shows the result of the optimal location and sizing of the DGP and the value of the objective function when installing these plants in the best buses of the system.

Electrical system under study

The test system was used by Baran and Wu in 1989 and is known in the literature as IEEE 69 bus [22]. It has allowed different authors [23]-[26] to validate algorithms that seek to integrate distributed generation optimally in electrical networks. The data of this distribution system were taken from [26].

Figure 6 shows the one-line diagram of this system [27]. The reference bus operates at a nominal voltage (12.66 kV), and the total installed active and reactive power load is 3801.9 kW and 2694.1 kvar, respectively.

Without installing distributed generation, the active power losses are 224.95 kW which represents 5.587% of the active power consumed by the system. The voltage of the worst bus is 0.9092 pu which represents a difference of 9.08% concerning the nominal voltage.

Figure 8 shows the execution of the CSM for the best assignment found when installing two plants operating at unity power factor, in the IEEE 69-buses system. The red dots are the best results obtained from the multi-objective function in each iteration of the algorithm, and the black square is the optimal solution to the problem. Please consider that you always move forward in one dimension by gradually decreasing the value of the multi-objective function. A more detailed analysis allows us to identify that the search in iterations 9 and 10 was executed with a shorter step. This iteration was accomplished to guarantee the convergence of the method towards the optimum point of the optimization problem.

Validation of the proposed algorithm

The algorithm was validated using the results obtained by other authors with a method called Elephant Herding Optimization (EHO) [26]. The optimal capacity of a DGP operating at unity

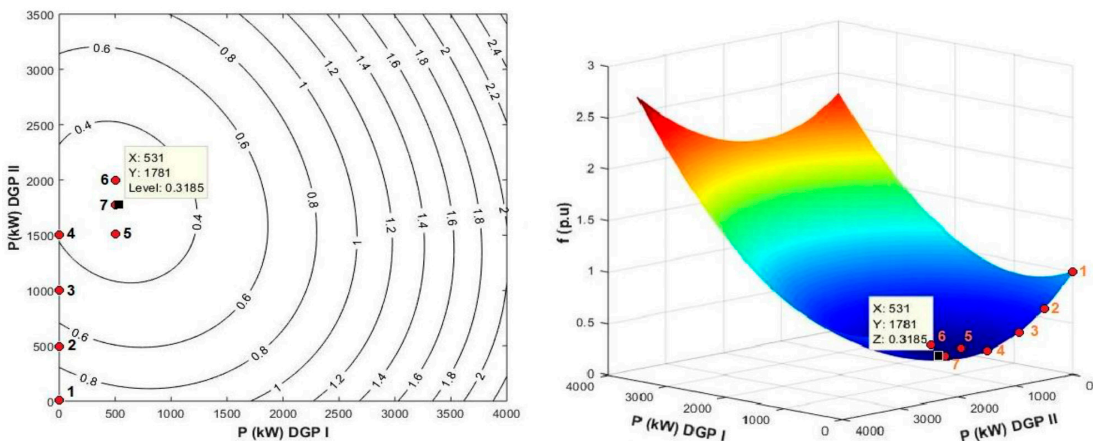


Figure 8. Example of execution of the CSM to reach the minimum of a function f .

power factor or 0.9 head was found using this method in a scenario that only sought to reduce active power losses with voltage constraints from 0.9 to 1.05 pu.

From this scenario, the results of the EHO method are compared in Table 1 with those of the algorithm proposed in this work. The differences between the capacities of the plants obtained in both methods do not exceed 0.4%, while the values of the active and reactive power losses differ only by 0.05%. The minimum voltages are the same except for the base case.

The accuracy and validity of the COCSM algorithm can be checked by analyzing the results of other methods, such as the PSO and the whale optimization algorithm, for the same optimization problem presented [23]-[29].

RESULTS AND DISCUSSIONS

A general scenario can be defined to facilitate the description of the algorithm results on which all the simulations to be performed are based. Two plants will be installed to generate active and reactive power at a power factor 0.9 in leading all system buses' minimum and maximum capacity restraints will be 0.5 MW and 2.5 MW, respectively. These restrains guarantee that all the installed power plants have a capacity that is between these values. The maximum deviation that the voltages in the buses can suffer will be $\pm 5\%$, and the thermal limits of the system feeders will be considered through their ampacity. Modifications to at least one of these parameters, which will be clarified before describing the results obtained, are necessary to demonstrate the algorithm's potential.

Influence of the value of θ

From equation 24, it can be seen that the multi-objective function depends on the weight factor (θ). If one wants to find the optimal solution to the problem, the value of this factor cannot be chosen arbitrarily. The evaluation strategy used to evaluate θ was to obtain Pareto frontiers [30] by installing one to four DGPs that perform reactive compensation. The objective functions used to construct these limits were the F_{AP} and F_{AU} variables shown in equation 22 and equation 23. These were expressed per unit based on initial losses (ΔP_0) and initial voltage deviation (ΔU_0). Figure 9 highlights the chosen Pareto-optimal solutions, as they minimize the multi-objective function in each scenario. Each of these solutions was obtained when the value of θ was 0.49. Table 2 also shows the results of the optimization algorithm with one and two DGPs when the parameter θ varies between 0.45 and 0.55. It should be mentioned that when the θ value is 0.49, the values given in the Pareto frontiers are obtained. It can be concluded that this is the value that should be assigned to the weight factor of the multi-objective function.

Influence of the number of DGP to install

The COCSM algorithm allows installing as many DGPs as buses in the distribution system. Table 3 shows the results of the method when locating and sizing from one to four DGPs in the 69-bus system, using 0.49 as the weight factor value.

Table 3 shows how the value of the objective function (F) decreases as the number of power plants installed in the system increases. These changes are consistent with the reduction in active power losses and the improvement in the voltage profile. Remarkably, the benefits obtained become

Table 1. Comparison between the results of the COCSM algorithm and the EHO.

IEEE 69	Without DGP		1 DGP $\cos(\varphi) = 1$		1 DGP $\cos(\varphi) = 0.9$ lead	
	COCSM	EHO	COCSM	EHO	COCSM	EHO
Location	–	–	61	61	61	61
DGP capacity (kVA)	–	–	1872.60	1873.60	2207.33	2216.30
Active power losses (kW)	224.959	225.000	83.188	83.230	27.940	27.950
Reactive power losses (kvar)	102.147	102.109	40.521	40.540	16.468	16.460
Bus with worst voltage	65	–	27	–	27	–
Worst voltage bus in pu	0.9092	0.9093	0.9683	0.9683	0.9724	0.9724

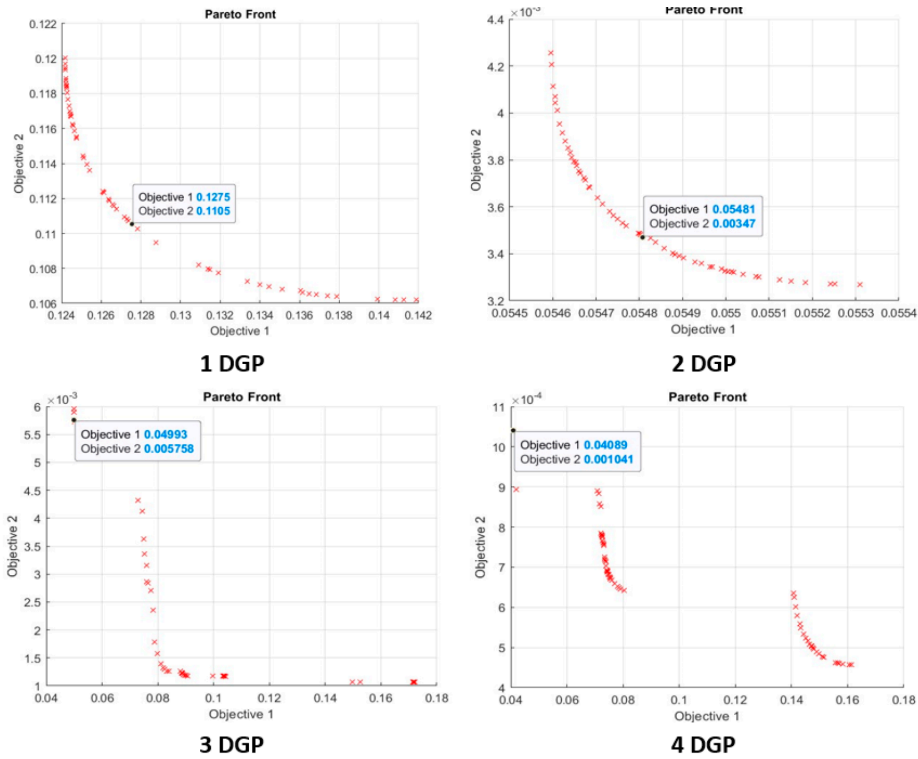


Figure 9. Pareto frontiers when installing one to four DGPs.

Table 2. Values of $F_{\Delta P}$ and $F_{\Delta U}$ when θ varies from 0.45 to 0.55.

1 DGP				2 DGP		
θ	$F_{\Delta P}$	$F_{\Delta U}$	$F_{\Delta P} + F_{\Delta U}$	$F_{\Delta P}$	$F_{\Delta U}$	$F_{\Delta P} + F_{\Delta U}$
0.45	0.12797	0.11013	0.23810	0.05469	0.00367	0.05837
0.46	0.12797	0.11013	0.23810	0.05469	0.00367	0.05837
0.47	0.12797	0.11013	0.23810	0.05469	0.00367	0.05837
0.48	0.12780	0.11029	0.23809	0.05469	0.00367	0.05837
0.49	0.12764	0.11044	0.23808	0.05469	0.00367	0.05837
0.50	0.12733	0.11076	0.23809	0.05469	0.00367	0.05837
0.51	0.12733	0.11076	0.23809	0.05470	0.00367	0.05837
0.52	0.12725	0.11084	0.23809	0.05469	0.00367	0.05837
0.53	0.12703	0.11109	0.23812	0.05509	0.00408	0.05917
0.54	0.12696	0.11117	0.23813	0.05568	0.00471	0.06039
0.55	0.12685	0.11130	0.11986	0.05634	0.00534	0.06168

smaller and smaller as the number of plants in the distribution system increases. It is also observed that when installing three and four DGPs, the minimum capacity constraint acts, limiting two to 500 kW in each case. Additionally, it is observed how the times increase exponentially as the number of plants installed increases. The most influential

factor is the number of buses considered by the program. The delay is quite significant when more than two plants are to be located and sized, taking 60320.873 seconds (approximately 16 hours, 45 minutes, and 21 seconds) for four plants. This delay is considered the major drawback of the exhaustive search method used in this work.

Table 3. Results when installing one to four DGPs in the IEEE 69-buses system.

IEEE 69	Scenarios				
	Without DGP	1 DGP	2 DGP	3 DGP	4 DGP
Location (bus)	–	61	17 & 61	10, 17 & 61	10, 17, 50 & 61
P (kW)	–	2133.6	576.8 1909.6	500 500 1807.2	500 500 746.2 1820.8
$\cos(\varphi)$ (all DGP)	–	0.900	0.900	0.900	0.900
Value of F	1.000	0.1189	0.0287	0.0223	0.0177
Active Power Losses (kW)	224.959	28.714	12.304	9.8248	7.6149
% reduction in active power losses	0.000	87.236	94.531	95.633	96.615
U_{min} (pu)	0.9092	0.9734	0.9942	0.9943	0.9980
U_{max} (pu)	1.000	1.0061	1.0013	1.0031	1.0032
Execution time (s)	–	1.338	43.793	1896.267	60320.873

Influence of maximum voltage and current restrictions

In the first scenario, two DGPs were installed, defining the minimum voltage restraint for all buses in the system at 0.99 per unit. In the second scenario, this restraint was adjusted to 1.00 per unit, although to comply with it was necessary to violate the bus capacity, maximum current, and plant operation (power factor) restraints. Also note the consequences of defining such strict restrictions since the value of the objective function is nine times higher than that of the initial state, and the system losses are 1671.04% higher. In scenarios I and II, the value of 1.05 per unit is maintained as the maximum voltage limit.

In the third scenario, the value of 1.00 ampere was set as the maximum current limit on lines 9-53 of the test system. The purpose of defining such a critical restraint is to show the ability of the algorithm to find solutions under these circumstances. The last scenario was modeled by forcing the current flowing through lines 4-47 to be less than 1.00 amperes. Initially, such lines were among the most heavily loaded in the system; a solution to these scenarios required a departure from the plant's operating restrictions. The results of the algorithm under restraints are shown in Table 4. These results demonstrate the advantages of this optimization method if very critical restraints need to be satisfied.

Table 4. Influence of voltage and current restrictions on the 69-buses IEEE system.

IEEE 69	Scenarios				
	Without DGP	I	II	III	IV
Location	–	17 & 61	5 & 62	12 & 57	48 & 57
P (kW)	–	576.8 1909.6	111903.4 1037.6	1039.5 1787.4	866.7 1788.1
$\cos(\varphi)$	–	0.900	0.7035 0.5350	0.8451 0.8177	0.8116 0.8178
Value of F	1.000	0.0287	8.8044	0.1550	0.2634
Active Power Losses (kW)	224.959	12.304	3984.37	61.003	74.794
% reduction in active power losses	0.000	94.531	1771.15	72.883	66.752
U_{min} (pu)	0.9092	0.9942	1.0000	0.9730	0.9647
U_{max} (pu)	1.000	1.0013	1.0421	1.0051	1.0000
Currents on lines 4-47 and 9-53 (A)	47.779 105.24	47.775 16.348	47.500 35.603	47.773 1.0000	1.0000 0.9991

In Figure 10, the voltage profiles of scenarios I and II are compared with those of the system's initial state. In the second scenario, the minimum voltage is 1 per unit; however, the mean square deviation of the voltage profile increases considerably concerning scenario I. Consequently, this leads to the conclusion that establishing such critical restraints in this type of optimization problem is only sometimes profitable.

Figure 11 shows the absolute value of the currents that circulate through some feeders of the system and the values of the maximum current restriction (I_{max}) for scenarios III and IV.

Figure 11 shows how the currents through all lines are below the established current restrictions. The currents through lines 4-47 and 9-53 are limited to almost 1 A in scenario IV.

Influence of the operating mode of DG plants

The proposed optimization algorithm allows modeling the generation or consumption of reactive power by the DGPs, limiting their operating power factor. If the reactive power generated by the capacitive effect of the lines is neglected, better results are obtained when the plants operate supplying reactive power. The distribution network was modified by installing three 600 kvar capacitor banks to allow at least one of the plants to operate with reactive power. The capacitors are located at buses 9, 19, and 61 of the network, reducing active power losses by 27.962% and increasing the voltage of bus 65 (the bus with the worst voltage) from 0.9092 pu to 0.9234 pu (Table 5).

Two scenarios are considered, taking as a starting point the system conditions with the capacitors

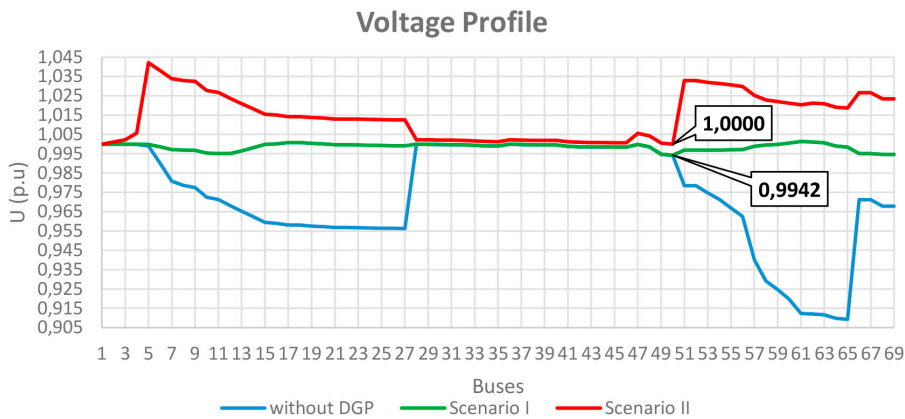


Figure 10. Comparison of IEEE 69-buses system voltage profiles obtained by installing two DGP that will guarantee the voltage restrictions of the COCSM algorithm.

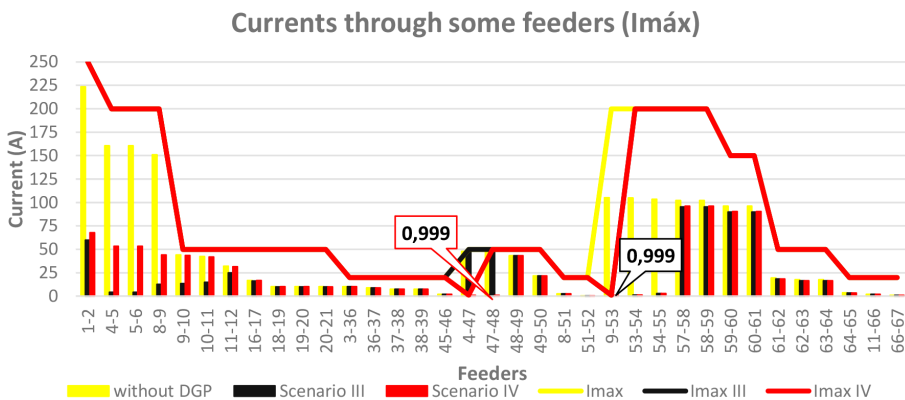


Figure 11. Comparison between the current modules and the maximum current restrictions of the IEEE 69-buses system conductors, in scenarios III and IV.

installed, in which the plants will operate in different regimes. In the first scenario, the capacities and locations of two plants that generate only active power will be found, while in the second, both plants can generate or consume reactive power without the power factor falling below 0.9.

Table 5 shows that the power plant installed at bus 19 consumes reactive power in the second scenario at a power factor at its operating limit. This means it compensates for generating the fixed capacitor bank at this bus. Furthermore, the DGP at bus 61 supplies reactive power together with the capacitor bank installed at the same bus. This analysis leads to the conclusion that the capacitor bank installed at bus 19 could have had a lower capacity (about 300 kvar), and the one at bus 61 could have had a higher capacity (about 750 kvar).

Figure 12 shows that in medium voltage distribution systems, both the generation of active power and the consumption or generation of reactive power allow the voltage profile to be modified.

Scenarios I and II significantly reduce the differences in bus voltages concerning the nominal value, thanks in part to their active power generation. Considering the voltages in the section comprising busbars 13 to 19 in scenario I, it can be observed that the excess reactive power due to the capacitor bank of bus 19 makes the deviation concerning the nominal voltage slightly higher than in scenario II. Similarly, in the section formed by buses 55 to 65, it can be seen how the slight deviation of the voltages in scenario I, regarding the nominal value, is compensated if the plant installed in bus 61 supplies reactive power.

Table 5. Influence of the DGP operating power factor on the results of the COCSM algorithm.

IEEE 69	Scenarios			
	Without DGP and Capacitors	With capacitors	I	II
Location	–	9,19 & 61	16 & 61	19 & 61
P (kW)	–	600 kvar	519.6 1846.4	526.4 1755.7
$\cos(\varphi)$	–	0	1.000 1.000	0.900 (lag) 0.938 (lead)
Value of F	1.000	–	0.0638	0.0247
Active Power Losses (kW)	224.959	162.055	20.000	7.5272
% reduction in active power losses	0.000	27.962	87.659	95.355
U_{min} (pu)	0.9092	0.9234	0.9941	0.9943
U_{max} (pu)	1.000	1.000	1.0026	1.0026

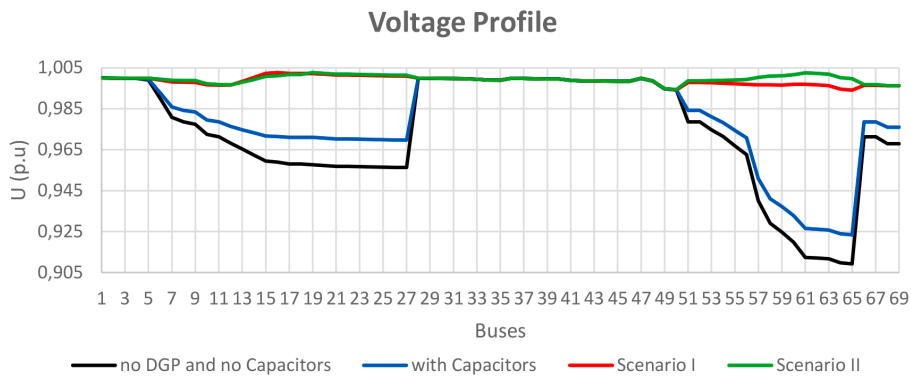


Figure 12. Comparison of the voltage profiles of the 69-buses system when installing capacitor banks and distributed generation (DGP) operating at different power factors.

CONCLUSIONS

This work used a hybrid method to solve a multiple-objective optimization problem. The purpose was the sizing and location of distributed generation plants to reduce the active power losses of a distribution system and improve its voltage profile. The proposed algorithm (COCSM) can obtain satisfactory results in systems whose restrictions are critical. It allows any distributed generation to be modeled in a steady state since its mode of operation can be adjusted to adapt it to any type of technology. The method also differs from others present in the bibliography because it does not use the generation of random numbers or a population to perform the search for the optimum of the objective function. Its core disadvantage is its execution time, which increases as the number of plants to be installed and the system buses to be analyzed rise. Consequently, it is proposed for future research to apply another method, such as PSO or EHO, to perform a less precise but fast search for the optimum, with which COCSM could obtain a fine-tuning of the capacities of the distributed generation plants.

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